Midterm 1
Physics 1B

Name:

ID number:

Lecture (1 or 2): Section:

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1. A rectangular flat-bottom barge with a bottom area $A = 100\,\text{m}^2$ is loaded so that the bottom is at $H = 1$ m below the surface. The density of water is $\rho = 10^3$ kg/m^3, and the water surface is perfectly still.

(a) Calculate the mass of the barge.

Solution. The force of buoyancy is equal and opposite the force of gravity:

$$Mg = V\rho g = AH\rho g. \tag{1}$$

Therefore, the mass of the barge M is given by

$$M = AH\rho = 100\,\text{m}^2 \times 1\,\text{m} \times 10^3\ \text{kg/m}^3 = 10^5\ \text{kg}\ . \tag{2}$$

(b) A round hole with radius $r = 2$ cm is made in the bottom of the barge, and the water starts leaking in. When the water level reaches $h = 5$ cm, a bilge alarm will alert the barge operator. How long will it take for the water to reach the level 5 cm? (Assume that the Bernoulli's equation is applicable.)

Solution. The Bernoulli's equation is

$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{const}\ . \tag{3}$$

Let us consider two points: the point at the water level outside the barge, and the point at the hole in the barge. We have

$$p_{\text{atm}} + \rho g H = p_{\text{atm}} + \frac{1}{2}\rho v^2\ , \tag{4}$$

so that

$$v = \sqrt{2gH}\ . \tag{5}$$

Now the volume that flows through the hole in time t is given by

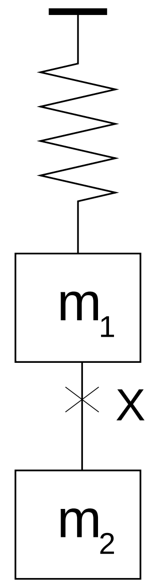
$$V = \pi r^2 v t\ . \tag{6}$$

This must be equal to Ah if the water is to fill the barge up to h, and so the time needed for that to happen is given by

$$t = \frac{Ah}{\pi r^2 v} = \frac{Ah}{\pi r^2\sqrt{2gH}} = \frac{100\ \text{m}^2 \times 0.05\ \text{m}}{\pi \times (0.02)\ \text{m}^2 \times \sqrt{2 \times 9.8\ \frac{\text{m}}{\text{s}^2} \times 1\ \text{m}}} = \tag{7}$$

$$= 898.17\ \text{s} \approx 15\ \text{min}\ . \tag{8}$$

2. The two weights, $m_1 = 2$ kg and $m_2 = 1$ kg hang on the spring as shown in the figure. Initially the system is in equilibrium, and the weights are at rest. When the string connecting the two weights is cut at point X, the lower one falls, and the upper one begins to oscillate with an amplitude $A = 0.2$ m. What is the period of these oscillations?



Solution. The upper weight starts the oscillation at rest, which means the amplitude A is equal the initial displacement. This initial displacement is caused by the force $(m_2 g)$, and, therefore,

$$kA = m_2 g \;\Rightarrow\; k = \frac{m_2 g}{A}$$

The period of oscillations is

$$T = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1 A}{m_2 g}} = 0.6s$$

3. A horizontal wire of length $L = 10$ m with a mass of 10 g is under tension. A transverse wave for which the frequency is $f = 750$ Hz, the wavelength is $\lambda = 0.1$ m, and the amplitude is $A = 3.7$ mm is propagating on the wire.

(a) Calculate the force of tension in the wire.

Solution. The mass of the wire is $m = 0.01$ kg, thus its linear density is

$$\mu = \frac{m}{L} = 10^{-3}\ \frac{\text{kg}}{\text{m}}\ . \tag{9}$$

We know that the velocity of the wave fulfills both

$$v = \lambda f \quad \text{and} \quad v = \sqrt{\frac{F}{\mu}}\ . \tag{10}$$

Therefore

$$F = \lambda^2 f^2 \mu = 10^{-2}\ \text{m}^2 \times (750)^2\ \frac{1}{\text{s}^2} \times 10^{-3}\ \frac{\text{kg}}{\text{m}} = 5.63\ \text{N}\ . \tag{11}$$

(b) Calculate the maximal transverse speed of a point on a wire.

Solution. The wave function is given by

$$y(x,t) = A\cos(kx - \omega t) = A\cos\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)\ . \tag{12}$$

Then the transverse speed is

$$v(x,t) = \frac{\partial y(x,t)}{\partial t} = 2\pi f A \sin\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)\ . \tag{13}$$

The maximal transverse speed is then

$$v_{\max}(x,t) = 2\pi f A = 2\pi \times 750\ \frac{1}{\text{s}} \times 3.7 \times 10^{-3}\ \text{m} = 17.44\ \frac{\text{m}}{\text{s}}\ . \tag{14}$$

(c) Calculate the maximal transverse acceleration of a point on a wire.

Solution. The acceleration is given by

$$a(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -4\pi^2 f^2 A \cos\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)\ , \tag{15}$$

so that the value of the maximal transverse acceleration is

$$a_{\max}(x,t) = 4\pi^2 f^2 A = 4\pi^2 \times (750)^2\ \frac{1}{\text{s}^2} \times 3.7 \times 10^{-3}\ \text{m} = 8.22 \times 10^4\ \frac{\text{m}}{\text{s}^2}\ . \tag{16}$$

4. A bat navigates by emitting ultrasonic waves with frequency $f_0 = 70$ kHz and detecting reflected signals.

(a) As it approaches a wall, the bat detects a reflected frequency of $f_1 = 75$ kHz. What is the speed of the bat?

Solution. The wave arriving at the wall is going to have a frequency

$$f_{\rm wall} = f_0 \frac{v}{v - v_{\rm bat}} \ . \tag{17}$$

Then this wave will propagate back towards the bat, which will detect

$$f_1 = f_{\rm wall} \frac{v + v_{\rm bat}}{v} = f_0 \frac{v + v_{\rm bat}}{v - v_{\rm bat}} \ . \tag{18}$$

This is equivalent to

$$f_1 \left(v - v_{\rm bat}\right) = f_0 \left(v + v_{\rm bat}\right) \quad \Rightarrow \quad v_{\rm bat} \left(f_0 + f_1\right) = v \left(f_1 - f_0\right) \ , \tag{19}$$

and

$$v_{\rm bat} = v \frac{f_1 - f_0}{f_0 + f_1} = 340 \ \frac{\rm m}{\rm s} \times \frac{75 - 70}{75 + 70} = 11.74 \ \frac{\rm m}{\rm s} \ . \tag{20}$$

(b) A moth hears the bat's sonar and plans an evasive maneuver. From $t = 0$ to $t = 3$ s, the sound intensity goes up by $\Delta\beta = 0.6$ dB. What is the time $t_{\rm ETA}$ at which the bat would catch up with the moth if the velocity of the moth remain constant?

Solution. At times $t_1 = 0$ and $t_2 = 3$ s, the bat is at distances r_1 and r_2, respectively. The bat passes distance $(r_1 - r_2)$ in time $(t_2 - t_1)$, and it will pass the remaining distance r_2 in time $(t_{\rm ETA} - t_2)$. The speed of the bat can be written for these two intervals:

$$\begin{aligned} v &= \frac{r_1 - r_2}{t_2 - t_1} = \frac{r_2}{t_{\rm ETA} - t_2} \quad \Rightarrow \\ t_{\rm ETA} &= t_2 + (t_2 - t_1)\frac{r_2}{r_1 - r_2} = t_2 + \frac{t_2 - t_1}{(r_1/r_2) - 1} \end{aligned} \tag{21}$$

To find the ratio (r_1/r_2), let us use the difference in sound intensity:

$$\begin{aligned} \Delta\beta &= \beta_2 - \beta_1 = (10 \text{ dB}) \left(\log \frac{I_2}{I_0} - \log \frac{I_1}{I_0}\right) = (10 \text{ dB}) \left(\log \frac{I_2}{I_1}\right) = \\ & (10 \text{ dB}) \left(\log \frac{r_1^2}{r_2^2}\right) = (20 \text{ dB}) \left(\log \frac{r_1}{r_2}\right) \quad (22) \\ & \Rightarrow \quad (r_1/r_2) = 10^{\Delta\beta/(20 \text{ dB})} \quad (23) \end{aligned}$$

Let us substitute this into equation (21) to obtain

$$t_{\rm ETA} = t_2 + \frac{t_2 - t_1}{(10^{\Delta\beta/(20 \text{ dB})} - 1)} = 45 \text{ s}.$$

5. Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. They are separated by distance $L = 10$ m. The frequency of the waves emitted by each speaker is $f = 68$ Hz. The speed of sound is $v_0 = 340$ m/s.

(a) You place a microphone between the speakers, along a line connecting them, and determine that you are at a point of constructive interference. How far from the microphone is the closest point of destructive interference?

Solution. If the microphone is at the point of constructive interference, it means the waves arrive 'in phase'. Let us now move a distance Δx along the line connecting the speakers (things get a bit easier if you imagine that the flow of time has stopped for a moment). The phase of the wave moving to the right changes by $\Delta x(+k) = +2\pi\frac{\Delta x}{\lambda}$, while the phase of the wave moving the left changes by $\Delta x(-k) = -2\pi\frac{\Delta x}{\lambda}$. We know that in order for the destructive interference to occur, the phase difference between the waves has to be equal $\pm\pi$. Let us then see what Δx fulfills this condition:

$$2\pi\frac{\Delta x}{\lambda} - (-2\pi\frac{\Delta x}{\lambda}) = 4\pi\frac{\Delta x}{\lambda} = \pm\pi \ , \tag{24}$$

or simply

$$\Delta x = \pm\frac{\lambda}{4} \ . \tag{25}$$

The wavelength in our example is given by

$$\lambda = \frac{v_0}{f} = \frac{340 \ \frac{\mathrm{m}}{\mathrm{s}}}{68 \ \frac{1}{\mathrm{s}}} = 5 \ \mathrm{m} \ , \tag{26}$$

so that

$$\Delta x = \pm 1.25 \ \mathrm{m} \ . \tag{27}$$

(b) How many points of constructive interference occur between the speakers on the straight line connecting them? (The locations of the speakers do not count even if they are such points.)

Solution. For the constructive interference to occur, the difference between the phases of interfering waves must be equal to 0, or 2π, or better yet: in the most general approach we say that it must be equal to an integer multiple of the full angle: $n2\pi$.

The wave moving to the right is described by

$$y_R(x,t) = A\cos[kx - \omega t] \ . \tag{28}$$

The wave moving to the left is described by

$$y_L(x,t) = A\cos\left[xk + \omega t\right] = A\cos\left[-kx - \omega t\right] \ . \tag{29}$$

At any given t and at any given x, the difference between the phases of these two waves is

$$[kx - \omega t] - [-kx - \omega t] = 2kx \ . \tag{30}$$

We know that for a constructive interference to occur, this difference must be equal $n \cdot 2\pi$. Explicitly,

$$2kx = 2\frac{2\pi}{\lambda}x = 2n\pi \ , \tag{31}$$

that is

$$x = n\frac{\lambda}{2} \ . \tag{32}$$

We know that in our example x must fulfill $0 < x < L$, or explicitly, we must have

$$0 < n \times 2.5 \text{ m} < 10 \text{ m} \ . \tag{33}$$

This is fulfilled for

$$n = 1 \ , \ 2 \ , \ 3 \ , \tag{34}$$

and therefore we have 3 points of constructive interference.

(c) How many points of destructive interference occur between the speakers on the straight line connecting them? (The locations of the speakers do not count even if they are such points.)

Solution. The way leading to this solution is the same as in part (b), only that for a destructive interference to occur we need a phase difference of $(2n-1)\pi$, that is we have a condition that

$$2\frac{2\pi}{\lambda}x = (2n-1)\pi \ , \tag{35}$$

or just

$$x = \left(n - \frac{1}{2}\right)\frac{\lambda}{2} \ . \tag{36}$$

Easily, this leads to

$$0 < \left(n - \frac{1}{2}\right) \times 2.5 \text{ m} < 10 \text{ m} \ , \tag{37}$$

which is fulfilled for

$$n = 1 \ , \ 2 \ , \ 3 \ , \ 4 \ . \tag{38}$$

and therefore we have 4 points of destructive interference.