MT2, L2

note: $K_e \equiv \frac{1}{4\pi\varepsilon_0}$

1. A non-conducting/insulating sphere of radius R is filled uniformly with charge of density ρ.

10 (a.) Find the electric field at points inside the sphere ($r \leq R$).

by symmetry, $\vec{E}$ is $\boxed{\text{radial}}$, (2) direction

$E(4\pi r^2) = 4\pi k_e \cdot Q_{encl}$,

$Q_{encl} = \rho \, 4/3 \pi r^3$

$$\Rightarrow \boxed{E_1 = 4/3 \pi k_e \rho r} \quad (8)$$

10 (b.) Find the electric field at points outside the sphere ($r > R$).

by symmetry, $\vec{E}$ is $\boxed{\text{radial}}$, (2)

$E(4\pi r^2) = 4\pi k_e \cdot Q_{total}$,

$Q_{total} = \rho \, 4/3 \pi R^3$

$$\Rightarrow \boxed{E_2 = k_e \frac{Q_{total}}{r^2}} \quad (8)$$

10 (c.) Find the voltage difference between infinity and the center of the sphere, $V(r = 0) - V(r \to \infty)$.

$$V_0 - V_\infty = -\int_\infty^0 E \, dr$$

$$= -\int_\infty^R E_2 \, dr - \int_R^0 E_1 \, dr$$

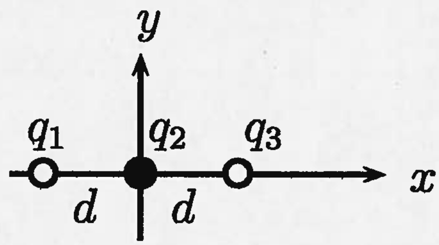
$$= \boxed{k \frac{Q_{total}}{R} \; (5) + 2/3 \pi k_e \rho R^2 \; (5)}$$

or

$$= \boxed{2\pi k_e \rho R^2}$$

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

2. Three point charges are positioned as shown in the Figure, with $q_1 = q_3 = -e$ and $q_2 = +2e$. The distance between adjacent charges is d. Ignore gravity.



10

(a.) Find the work done by an external agent to assemble the system.

$$W = k_e \frac{q_1 q_2}{d} + k_e \frac{q_1 q_3}{2d} + k_e \frac{q_2 q_3}{d}$$

$$= k_e \frac{-2e^2}{d} + k_e \frac{e^2}{2d} + k_e \frac{-2e^2}{d}$$

$$= -\tfrac{7}{2} k_e \frac{e^2}{d}$$

10

(b.) Find the electric potential at an arbitrary point on the y-axis.

$$V = k_e \frac{-e}{\sqrt{y^2+d^2}} + k_e \frac{2e}{|y|} + k_e \frac{-e}{\sqrt{y^2+d^2}}$$

$$= -2 k_e \frac{e}{\sqrt{y^2+d^2}} + k_e \frac{2e}{|y|}$$

note: $|y| = \sqrt{y^2}$

10

(c.) Find the electric field at an arbitrary point on the y-axis. Give all components, E_x, E_y, and E_z.

5

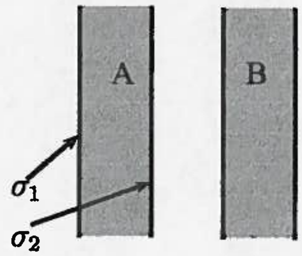
by symmetry, $E_x = E_z = 0$ $\Big\{$ Part b) may not be used to find E_x & E_z

$$E_y = -\frac{\partial V}{\partial y} \overset{5}{=} -2k_e \frac{e y}{(y^2+d^2)^{3/2}} + k_e \frac{2e y}{|y|^3}$$

note: $-\frac{\partial}{\partial y} \frac{1}{\sqrt{y^2}} = \frac{y}{(y^2)^{3/2}} = \frac{y}{|y|^3}$

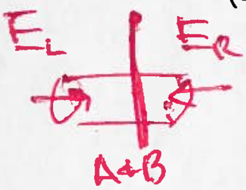
$K = \frac{1}{4\pi\varepsilon_0}$

3. Consider two infinite conducting plates. Plate A has a total surface charge -4σ and Plate B has total surface charge -2σ. The plates are parallel and initially isolated from each other.



10

(a.) Find the magnitude of electric field outside and between the plates.



$\vec{E}_L = -\vec{E}_R$, Gauss's Law :

$-2E_L = 4\pi k \cdot (-6\sigma)$

$\boxed{E_L = 12\pi k \sigma}$ 5

Gauss's Law :

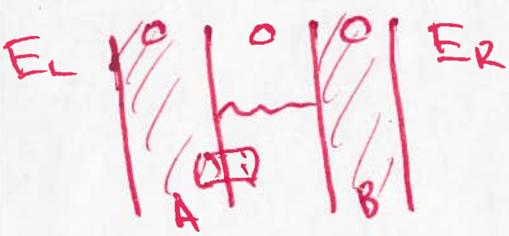
$-E_L - E = 4\pi k(-4\sigma) \Rightarrow \boxed{E = 4\pi k \sigma}$ 5

10 (b.) What are the charge densities, σ_1 and σ_2, on the surfaces of Plate A?

$-E_L + 0 = 4\pi k \sigma_1$

$\Rightarrow \boxed{\begin{array}{l} \sigma_1 = -3\sigma \\ \sigma_2 = -\sigma \end{array}}$ 5 5 by charge cons.

10 (c.) The plates are now connected to each other with a small conducting wire. Find the new charge densities on both surfaces of Plate A.



A+B are now equipotential.

$\Rightarrow$ zero E between A+B.

$\Rightarrow \sigma_2 = 0$ 5

E_L unchanged $\Rightarrow \sigma_1 = -3\sigma$ 5