Physics 1B
Midterm Exam 2

Spring 2013, UCLA, A. Forrester

Full Name (printed) Solutions

Full Name (signature) ______________________

Student ID Number ______________________

Seat Number ______________________

Problem	Grade
1 (a–f)	/10
2 (a–c)	/25
3	/15
Total	/50

- Do not peek at the exam until you are told to begin. You will have approximately the whole class period (50 minutes) to complete the exam.
- You are allowed one 3"×5" card of notes (both sides), but all other books or notes must be put away. A calculator won't be necessary; you must put them away too.
- Show your work to get full credit. (Exception: You don't need to show work in the multiple-choice problems. Brief sentence fragments or equations will do for the short-answer questions.)
- For a 100% score, budget your time roughly by 1 minute per point (50 pts for 50 min). Otherwise just do your best and finish what you can. You can move on if a problem is taking you too long; you will get partial credit for partial solutions you leave behind.

1. *A collection of problems.*

1a) (1 point) If a particle with known charge q is placed at a point where $\mathbf{E}$ is known (and the charge is not large enough to disturb the system that creates the electric field in that region), can the electric force on that charge be determined? If so, how? If not, why not?

Yes. $\vec{F}_E = q\vec{E}$ (by definition of $\vec{E}$)

1b) (2 points) If the electric potential at a single point is known, can $\mathbf{E}$ at that point be determined? If so, how? If not, why not?

No. $\vec{E} = -\vec{\nabla} V$ (To take a derivative you need the value of V for immediately surrounding points.)

1c) (2 points) What following factors below are necessarily involved in polarization-induced-by-a-charge and the-attraction-between-the-polarized-object-and-the-inducing-charge? (Circle all that apply.)

polarization

A. Mobility of charge in matter – there is generally some amount of mobility of charge, even if it is "bound" and can only move within a short range.

B. In (quasi)electrostatics, the charge in a conductor is found on the surfaces of the conductor, distributed in such a way as to cancel the electric field inside the material of the conductor.

C. A closer charge has a greater effect than a further-away charge.

D. Like-signed charges repel.

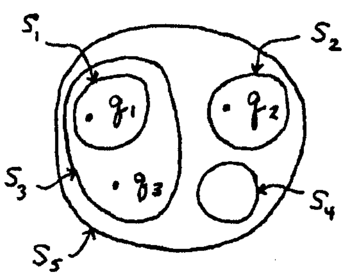
E. Unlike-signed charges attract.

attraction

1d) (2 points) Does induced-polarization always imply an attraction to the inducing charge(s)? If so, why? If not, why not?

No. Uniform $\vec{E}$ field $\Rightarrow$ no net force (could be a net torque, though)

1e) (2 points) As shown below, three charges lie in a plane. Let $q_1 = 1\ \mu\text{C}$, $q_2 = -2\ \mu\text{C}$, and $q_3 = 3\ \mu\text{C}$. Five closed surfaces are drawn, with each enclosing certain charges (or no charges). Only the intersection of these surfaces with the plane is shown. Rank these surfaces (S_1, S_2, S_3, S_4, S_5) by the electric flux on each surface, from greatest to least flux (or "most positive" to "most negative" flux).



$S_1 \to 1\,\mu C/\varepsilon_0$
$S_2 \to -2\,\mu C/\varepsilon_0$
$S_3 \to 4\,\mu C/\varepsilon_0$
$S_4 \to \varnothing$
$S_5 \to 2\,\mu C/\varepsilon_0$

(most flux) 1. S_3
2. S_5
3. S_1
4. S_4
(least flux) 5. S_2

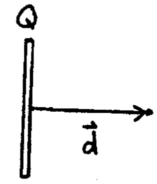
1f) (1 point) If you have a conductor that has a cavity with charges inside it, do you have to do anything in order to shield the outside beyond the conductor from electrostatic effects due to charges in the cavity? If so, what do you have to do? If not, why not?

Yes. Ground (the outer surface of) the conductor.

2. *Square geometries.*

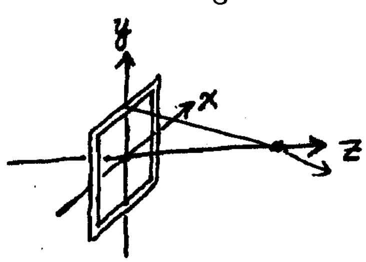
In class we examined a finite line of charge of length ℓ with uniform linear charge density and total charge Q. We found that the electric field due to this line charge at a point a (perpendicular) displacement $\mathbf{d}$ away from the center of the line is

$$\mathbf{E} = \frac{kQ}{d}\frac{1}{\sqrt{d^2+(\ell/2)^2}}\hat{\mathbf{d}}.$$

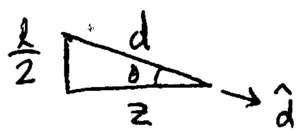


(In class we used the letter x instead of d, and b instead of $\ell/2$.) Use this formula to solve the following problems.

2a) (10 points) Consider a square loop of charge with uniform linear charge density and total charge q, pictured below. The loop is in the xy-plane, centered at the origin, and each side of the loop has length ℓ. Calculate the electric field at any point on the z-axis due to this charge.



Fields of 4 line segments add
only z-component remains
(x and y components cancel)



$d = \sqrt{z^2 + (l/2)^2}$

$\cos\theta = \frac{z}{d}$

$$\vec{E} = 4 E_z' \hat{z}$$

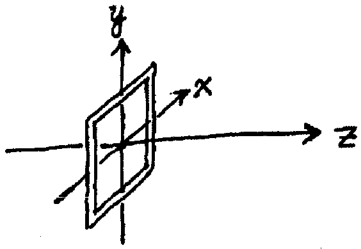
$$= 4 E' \cos\theta \, \hat{z}$$

$$= 4\left(\frac{k\frac{q}{4}}{d}\frac{1}{\sqrt{d^2+(l/2)^2}}\right)\left(\frac{z}{d}\right)\hat{z}$$

$$= \frac{kqz}{d^2}\frac{1}{\sqrt{d^2+(l/2)^2}}\hat{z}$$

$$= \frac{kqz}{(z^2+(l/2)^2)}\frac{1}{\sqrt{z^2+2(l/2)^2}}\hat{z}$$

2b) (7 points) Suppose a point charge $-q$ of mass m is constrained to move along the z-axis in the presence of the square loop of charge q. The point charge is placed at rest a displacement $z = z_0 \ll \ell$ from the origin. After it is released, what will be the frequency of its oscillation?



q and $-q$ $\Rightarrow$ attraction (to origin)

$z << \ell$

$$\vec{F}_E = -q\vec{E} = -\frac{kq^2 z}{(z^2 + (\ell/2)^2)} \frac{1}{\sqrt{z^2 + 2(\ell/2)^2}} \hat{z}$$

$$= -\frac{kq^2 z}{(\ell/2)^2} \frac{1}{\sqrt{2(\ell/2)^2}} \hat{z} \quad \text{since } z << \ell$$

$$= -\frac{kq^2 z}{\sqrt{2}(\ell/2)^3} \hat{z} = -\left(\frac{kq^2}{\sqrt{2}(\ell/2)^3}\right) z \hat{z}$$

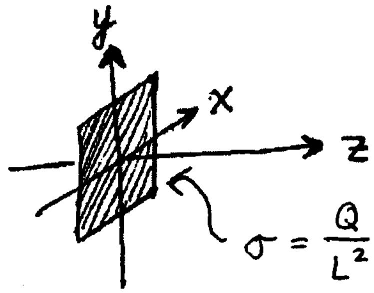
$$= k_{eff}\, z\, \hat{z}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{k_{eff}}{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{kq^2}{\sqrt{2}\, m\, (\ell/2)^3}}$$

(amplitude of oscillation is z_0)

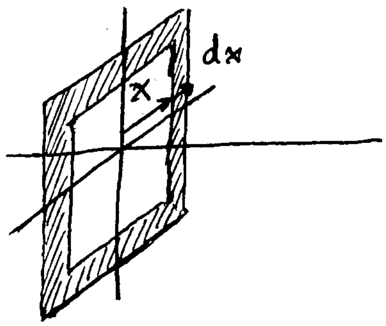
2c) (8 points) Consider a flat surface of finite size in the shape of a square, pictured below. The square has a uniform surface charge density with a total charge Q. The square is in the xy-plane, centered at the origin, and each side of the square has length L. Construct an integral expression for the electric field at any point on the z-axis due to this charge. (You don't need to solve the integral, but do pull constant factors out of the integral.)



Use square-loop result with

$q = dQ$ and $\ell = 2x$

and add up (integrate)...



$$\vec{E} = \int d\vec{E}$$

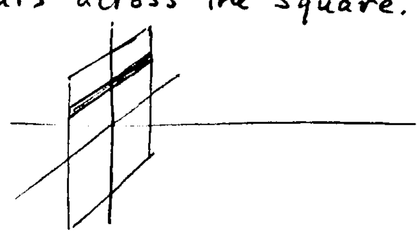
$$= \int \frac{k\, dQ\, z}{(z^2 + (\ell/2)^2)} \frac{1}{\sqrt{z^2 + 2(\ell/2)^2}}\, \hat{z}$$

$$= 8 \frac{kQ}{L^2} z \hat{z} \int_0^{L/2} \frac{x\, dx}{(z^2 + x^2)\sqrt{z^2 + 2x^2}}$$

$\ell = 2x$

$$dQ = \sigma\, dA$$
$$= \sigma\, 4\ell\, dx$$
$$= 8\sigma\, x\, dx$$
$$= 8 \frac{Q}{L^2} x\, dx$$

You can also use the original line result and integrate the effect of all bars across the square. Showing that the result is the same as that above is tricky, even with an online integrator.



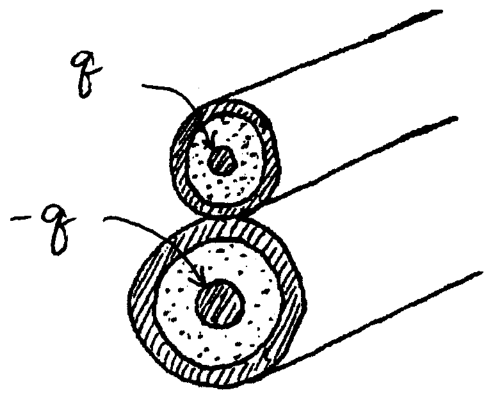
3. *Double-cylinder capacitor.*

(15 points) Consider the cylinders pictured below. Both cylinders are very long with the same length ℓ that is very large compared their diameters. For each cylinder, the central material and the outermost material are conducting, and the material between the conductors is dielectric with dielectric constant K. The two cylinders are touching along their whole length. For the top cylinder, the radii for each boundary are, from smallest to largest, R_1, R_2, and R_3. For the bottom cylinder these radii are R_4, R_5, and R_6.

Suppose some charge q is placed on the central conductor of the top cylinder and some charge $-q$ is place on the central conductor of the bottom cylinder.

- What is the capacitance of this arrangement?

Ignore end effects (i.e., the effects due to the fringing of the fields at the ends of the cylinders).



You can approach this as two capacitors in series or explicitly calculate the potential difference... $q = CV \Rightarrow C = \frac{q}{V}$

top: $\vec{E}_1 = \frac{1}{K}\vec{E}_{10} = \frac{\lambda_{in}}{K\,2\pi s\,\varepsilon_0}\hat{s} = \frac{q/\ell}{2\pi s\,\varepsilon}\hat{s}$

bottom: $\vec{E}_2 = \frac{-q/\ell}{2\pi s\,\varepsilon}\hat{s}$

$$V = V_{-q\to q} = -\int \vec{E}\cdot d\vec{\ell} = -\int_{R_4}^{R_5} \vec{E}_2\cdot d\vec{\ell} - \int_{``R_5"}^{``R_2"} \vec{\emptyset}\cdot d\vec{\ell} - \int_{R_2}^{R_1} \vec{E}_1\cdot d\vec{\ell}$$

across both conducting shells

$$= +\frac{q/\ell}{2\pi\varepsilon}\int_{R_4}^{R_5}\frac{ds}{s} - \frac{q/\ell}{2\pi\varepsilon}\int_{R_2}^{R_1}\frac{ds}{s} = \frac{q}{2\pi\ell\varepsilon}\left(\ln\left(\frac{R_5}{R_4}\right) - \ln\left(\frac{R_1}{R_2}\right)\right) = \frac{q}{2\pi\ell\varepsilon}\ln\left(\frac{R_2 R_5}{R_1 R_4}\right)$$

$$\Rightarrow \quad C = \frac{q}{V} = \frac{2\pi\ell K\varepsilon_0}{\ln\left(\frac{R_2 R_5}{R_1 R_4}\right)}$$