Physics 1B
Midterm Exam 1

Spring 2013, UCLA, A. Forrester

Full Name (printed) Solutions

Full Name (signature) ______________________

Student ID Number ______________________

Seat Number ______________________

Problem	Grade
1 (a–f)	/13
2	/12
3	/12
4	/13
Total	/50

- Do not peek at the exam until you are told to begin. You will have approximately the whole class period (50 minutes) to complete the exam.
- You are allowed one 3"×5" card of notes (both sides), but all other books or notes must be put away. You can use a calculator for calculations only.
- Show your work to get full credit.
- For a 100% score, budget your time roughly by 1 minute per point (50 pts for 50 min). Otherwise just do your best and finish what you can. You can move on if a problem is taking you too long; you will get partial credit for partial solutions you leave behind.

Table of Moments of Inertia

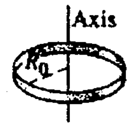
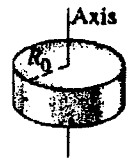
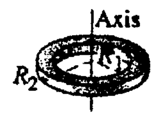
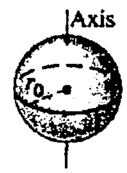
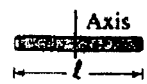
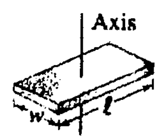
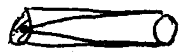
	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R_0	Through center		MR_0^2
(b)	**Thin hoop,** radius R_0 width w	Through central diameter		$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder,** radius R_0	Through center		$\frac{1}{2}MR_0^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius r_0	Through center		$\frac{2}{5}Mr_0^2$
(f)	**Long uniform rod,** length ℓ	Through center		$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod,** length ℓ	Through end		$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate,** length ℓ, width w	Through center		$\frac{1}{12}M(\ell^2 + w^2)$

FIGURE 10–20 Moments of inertia for various objects of uniform composition. [We use R for radial distance from an axis, and r for distance from a point (only in e, the sphere), as discussed in Fig. 10–2.]

1. *A collection of problems.*

1a) (2 points) When you blow air into an open organ pipe, it produces a sound with a fundamental frequency of 440 Hz.
If you close one end of this pipe and blow again, exciting a new fundamental mode, what frequency will you hear from the the sound emerging from the pipe?

open:  $L = \frac{\lambda}{2}$ $\quad f = \frac{v}{\lambda} = \frac{v}{2L}$

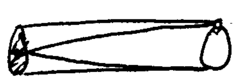
closed-open:  $L = \frac{\lambda'}{4}$ $\quad f' = \frac{v}{\lambda'} = \frac{v}{4L} = \frac{1}{2} f$

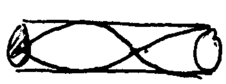
$$\Rightarrow \quad f' = \tfrac{1}{2}(440\ \text{Hz}) = 220\ \text{Hz}$$

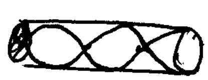
1b) (3 points) While playing with a (Kundt) tube with one closed end and one open end and little foam chips inside, you notice that when you play sound at a frequency of 300 Hz, the tube resonates and the chips collect in certain locations that indicate that there is a total of two displacement nodes.
What frequency should you play to create twelve displacement nodes?

Need to find relationship between freq. and # of nodes N.

$f = n \frac{v}{4L} = n f_1 \qquad n = \text{odd} = 1, 3, 5, \ldots$

 $n = 1 \qquad N = 1$

300 Hz  $n = 3 \qquad N = 2 \quad \longleftarrow$ so $f_1 = 100$ Hz

 $\underbrace{n = 5 \qquad N = 3}_{n = 2N - 1}$

12 nodes $\Rightarrow \quad n = 2(12) - 1 = 23 \qquad f = 23 f_1 = 23(100\ \text{Hz})$

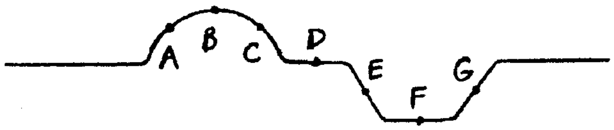
$= 2300$ Hz

1c) (1 point) A wave of a certain shape, shown below at an instant in time, travels along a string. At this instant, which labeled point or points have

- zero velocity? B, D, F
- zero acceleration? D, E, F, G

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

$$a_y = v^2 \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)$$



$\therefore$ changing slope $\Rightarrow$ accel.

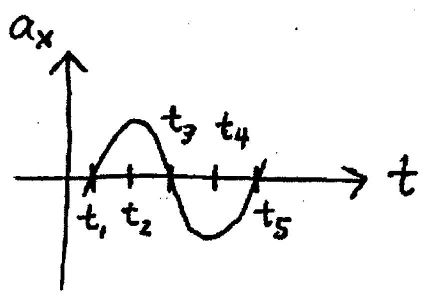
straight segment $\Rightarrow$ no accel. (const. vel.)

(It may help to think of the wave equation.)

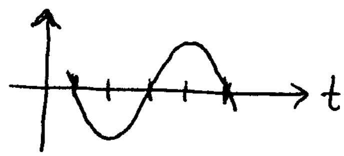
(zero slope $\Rightarrow$ zero vel.)

1d) (3 points) The graph below shows acceleration of a simple harmonic oscillator as a function of time, over a certain interval of time. Within the shown interval, at what time or times does the oscillator have

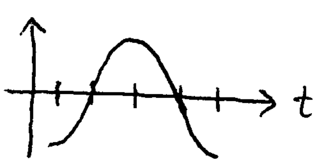
- the most positive displacement? t_4
- the most positive velocity? t_3
- the most kinetic energy? t_1, t_3, t_5



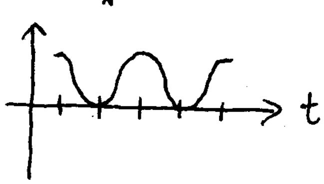
$x \propto -a_x$

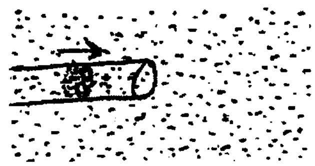


$v_x = dx/dt$



$K \propto v_x^2$

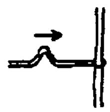




partially transmits
partially reflects inverted
(as low pressure pulse)

(A sound-wave pressure-pulse traveling through a tube.)

1e) (2 points) Consider a pulse of high pressure that travels through a tube toward an open end of the tube. Consider its behavior when it reaches the end of the tube.
Which picture below shows the analogous case for transverse displacement in a string? (It may help to consider the normal modes in terms of relative pressure $\Delta p = p - p_{\text{ambient}}$.)

A. $\Delta p \Rightarrow$  (free end)

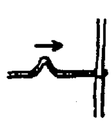
B. $\Delta p \Rightarrow$  (connected to lighter string)

C. $\Delta p \Rightarrow$  (no boundary)

D. $\Delta p \Rightarrow$  (connected to heavier string) partially transmits partially reflects inverted

E. $\Delta p \Rightarrow$  (fixed end)

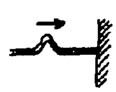
1f) (2 points) Again, consider a pulse of high pressure that travels through a tube toward an open end of the tube. But this time consider this phenomenon in terms of displacement of the air molcules.
Which picture below shows the analogous case for transverse displacement in a string? (It may help to consider the normal modes in terms of air displacement y.)

A. $y \Rightarrow$ 

B. $y \Rightarrow$ 

C. $y \Rightarrow$ 

D. $y \Rightarrow$ 

E. $y \Rightarrow$ 

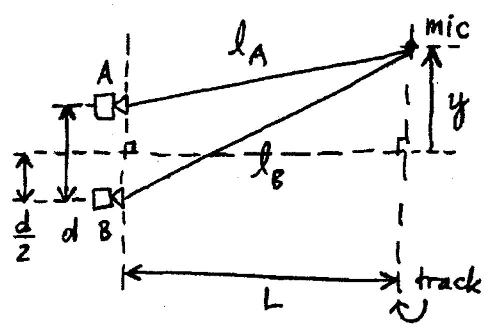
partially transmits
partially reflects upright

2. *Microphone recording sound.*

(12 points) Two speakers A and B each produce sinusoidal sound waves of frequency f that radiate in every direction. Speaker A is three quarters of a cycle out of phase (behind) speaker B. The speakers are a distance d apart, and they are both a distance L away from a track along which a microphone may slide. Let y be the position of the microphone on the track, as shown below.

- Using these variables, write an equation that (in principle) could be solved to find the positions for which the microphone records the least amount of sound.

  (You don't need to solve for these positions.)



least ⇒ destructive interference

$\Rightarrow \Delta N = n + \frac{1}{2}$ n = integer $(\ldots,-2,-1,0,1,2,\ldots)$

speaker A is $\frac{3}{4}$ cycle behind

⇒ wave A starts $\frac{3}{4}$ cycle ahead

$$N_A = \frac{3}{4} + \frac{l_A}{\lambda} \qquad l_A = \sqrt{L^2 + (y - \tfrac{d}{2})^2} \qquad \lambda = \frac{v}{f} \leftarrow \text{speed of sound}$$

$$N_B = \frac{l_B}{\lambda} \qquad l_B = \sqrt{L^2 + (y + \tfrac{d}{2})^2}$$

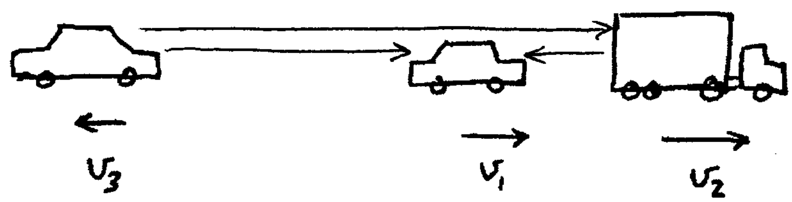
$$\Delta N = N_A - N_B = \frac{3}{4} + \frac{\Delta l}{\lambda}$$

$$= \boxed{\frac{3}{4} + \frac{f}{v}\left(\sqrt{L^2 + (y - \tfrac{d}{2})^2} - \sqrt{L^2 + (y + \tfrac{d}{2})^2}\right) = n + \frac{1}{2}}$$

3. *Hearing a car horn.*

(12 points) Take the speed of sound in air to be 344.0 m/s. Suppose you're driving a car at a speed of $v_1 = 25.00$ m/s and you've just pulled onto a highway with a truck moving ahead of you at a speed of $v_2 = 27.00$ m/s. Another car, traveling at $v_3 = 23.00$ m/s in the opposite direction, passes you and honks as it gets farther away. The horn has a frequency $f = 500.0$ Hz. The sound from the horn reaches your ear directly but also travels ahead of your car, reflects off the truck in front of you, and reaches your ear in this indirect manner.

- Assuming that the two sound waves that reach your ear have approximately the same amplitude, what is the frequency of the tone you hear and what beat frequency do you hear? (There is no wind. Be sure to keep four significant figures and round appropriately.)



direct: $$f_{o1} = \left(\frac{v_m + (-v_1)}{v_m - (-v_3)}\right) f = \left(\frac{v_m - v_1}{v_m + v_3}\right) f = 434.6 \text{ Hz}$$

indirect: (reflected) $$f_2 = \left(\frac{v_m + (-v_2)}{v_m - (-v_3)}\right) f = \left(\frac{v_m - v_2}{v_m + v_3}\right) f = 431.9 \text{ Hz}$$

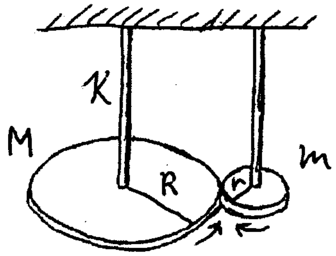
$$f_{o2} = \left(\frac{v_m + (v_1)}{v_m - (-v_2)}\right) f_2 = \left(\frac{v_m + v_1}{v_m + v_2}\right) f_2 = 429.6 \text{ Hz}$$

Heard: $$f' = \frac{f_{o1} + f_{o2}}{2} = 432.1 \text{ Hz}$$

$$f_{beat} = |f_{o1} - f_{o2}| = 5.1 \text{ Hz}$$

4. *Ceiling decoration.*

(13 points) See the diagram below for reference. A ceiling decoration consists of a circular disk of mass M and radius R (with a mural on its bottom) suspended from the ceiling by a cable connected to the center of the disk. When the disk rotates, the cable twists and provides a restoring torque with torsion constant κ to bring it back to its equilibrium orientation. An accessory piece allows the period to be altered: a smaller disk of mass m and radius r rotates without friction on a pole through its center, and its edge is brought into contact with the larger disk's edge so the disks rotate against each other without slipping. What's the period of oscillation of this decoration (with the accessory)?



rotation w/o slip: $R\,\Delta\theta_1 = r\,\Delta\theta_2$

$R\,\omega_1' = r\,\omega_2'$

$R\,\alpha_1 = r\,\alpha_2$

SHO Energy technique:

$$E = \tfrac{1}{2} I_1 \omega_1'^2 + \tfrac{1}{2} I_2 \omega_2'^2 + \tfrac{1}{2}\kappa\theta_1^2$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega_1'^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\left(\tfrac{R}{r}\omega_1'\right)^2 + \tfrac{1}{2}\kappa\theta_1^2$$

$$= \tfrac{1}{2}\left[\tfrac{1}{2}(M+m)R^2\right]\omega_1'^2 + \tfrac{1}{2}\kappa\theta_1^2$$

$$= \tfrac{1}{2} I_{eff}\,\omega_1'^2 + \tfrac{1}{2}\kappa\theta_1^2$$

$$\frac{dE}{dt} = I_{eff}\,\cancel{\omega_1'}\,\alpha_1 + \kappa\theta_1\cancel{\omega_1'} = 0$$

$$\Rightarrow \quad \alpha_1 = -\frac{\kappa}{I_{eff}}\theta_1 \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{\kappa}{I_{eff}}}$$

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I_{eff}}{\kappa}} = 2\pi\sqrt{\frac{\tfrac{1}{2}(M+m)R^2}{\kappa}}$$