Physics 1B
Final Exam

Spring 2013, UCLA, A. Forrester

Full Name (printed) Solutions

Full Name (signature) ____________________

Student ID Number ____________________

Seat Number ____________________

Problem	Grade
1 (a–v)	/39
2	/14
3 (a,b)	/17
4 (a–d)	/25
5	/15
6 (a–c)	/20
Total	/130

- Do not peek at the exam until you are told to begin. You will have approximately the whole exam period (3 hours) to complete the exam.
- You are allowed one 5"×7" card of notes (both sides), but all other books or notes must be put away. You can use a calculator for calculations only.
- Show your work to get full credit. (Exception: You don't need to show work in the multiple-choice problems. Brief sentence fragments or equations will do for the short-answer questions.)
- If you initially budget your time roughly by 1 minute per point (130 pts for 130 min), then you'll be able to judge where to spend the remaining 50 minutes you have left. Do your best and finish what you can. You can move on if a problem is taking you too long; you will get partial credit for partial solutions you leave behind.

1. *A collection of problems.*

1a) (1 point) A forced oscillator will oscillate at

A. the intrinsic, natural frequency of the oscillator.
[✓ B.] the frequency of the driving oscillator.
C. a frequency that depends on both the natural and driving frequencies.

1b) (1 point) A forced oscillator will oscillate with an amplitude that depends on

A. the natural frequency of the oscillator (but not the driving frequency).
B. the frequency of the driving oscillator (but not the natural frequency).
[✓ C.] both the natural and the driving frequencies.

1c) (1 point) A forced oscillator with small damping will resonate

A. at the natural frequency of the oscillator.
[✓ B.] near the natural frequency of the oscillator, but slightly lower.
C. near the natural frequency of the oscillator, but slightly higher.

1d) (1 point) (Circle all that apply.) For the simple superimposable waves we discussed in class, the speed of the wave is fundamentally determined by

A. the frequency of the wave.
B. the wavelength of the wave.
C. the amplitude of the wave.
[✓ D.] the elasticity and inertia of the medium.

1e) (2 points) Suppose the tension in a guitar string is reduced by a factor of 81%. By what factors do the following change:

- the string's wave speed: 90 %
- the frequency of the string's oscillation: 90 %
- the wavelength of the string wave: 100 % (no change)
- the generated sound's wave speed: 100 %
- the pitch (frequency) of the generated sound: 90 %
- the wavelength of the generated sound: 111 %

Same freq.

$T \to T' = 0.81T$

$v = \sqrt{\frac{T}{\mu}}$

$v \to \sqrt{0.81} = 0.90$

$v = \lambda f$

$(0.90) = (1.00)(0.90)$

$(1.00) = (1.\overline{11})(0.90)$

1f) (2 points) Suppose a room has an organ pipe that plays a note at a particular pitch and a guitar with a string that plays at the same pitch. If the air in the room is replaced with helium, how will the pitches of the sounds generated by the pipe and the string change? Same geometry/wavelengths

pipe: gas has less mass/inertia ⇒ higher wave speed
⇒ higher gas vibration freq. ⇒ higher sound freq/pitch

string: same string mass & tension ⇒ same string wave speed & vibration freq.
⇒ same sound freq./pitch

1g) (1 point) Suppose you bring a positive charge near (but not touching) a conductor, touch the conductor with your finger, hold your finger on the conductor as the charge is taken far away, and take your finger off the conductor. Then you bring a negative charge near (but not touching) the conductor. Is there a force between the conductor and the charge?

A. Yes, you have charged the conductor by induction and there is a repulsive force.

B. Yes, you have charged the conductor by induction and there is an attractive force.

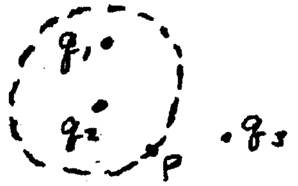
C. Yes, although you haven't charged the conductor, there is a repulsive force.

[✓ D.] Yes, although you haven't charged the conductor, there is an attractive force.

E. No, there is no force.

1h) (2 points) Three charges and a closed Gaussian surface are shown below, to the right.

- Which of the charges contribute to the electric field at point P? All 3.
- Which of the charges contribute to the net electric flux on the Gaussian surface? Only q_1 & q_2.



1i) (2 points) In which of the following cases does an electric dipole experience a non-zero net force? The dipole is in the field...

[✓ A.] of a point charge with the charge on the dipole axis.

B. within an ideal parallel plate capacitor with the dipole axis at 45° to the field.

C. of a thin, infinitely long charged rod with the dipole axis parallel to the rod.

[✓ D.] of a thin, infinitely long charged rod with the dipole axis 45° to the rod.

[✓ E.] of a thin, infinitely long charged rod with the dipole axis perpendicular to the rod.

F. of an infinitely large plane of charge with the dipole axis perpendicular to the plane.

1j) (2 points) In which of the following cases does an electric dipole experience a non-zero net torque? The dipole is in the field of...

A. of a point charge with the charge on the dipole axis.

[✓ B.] within an ideal parallel plate capacitor with the dipole axis at 45° to the field.

[✓ C.] of a thin, infinitely long charged rod with the dipole axis parallel to the rod.

[✓ D.] of a thin, infinitely long charged rod with the dipole axis 45° to the rod.

E. of a thin, infinitely long charged rod with the dipole axis perpendicular to the rod.

F. of an infinitely large plane of charge with the dipole axis perpendicular to the plane.

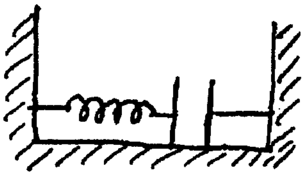
1k) (1 point) A capacitor is connected to a battery until charged and then it is disconnected. What happens when you increase the distance between the plates?

[✓ A.] The voltage across the plates increases.
B. The electric field between the plates decreases.
C. The magnitude of charges on the plates decrease.

1l) (1 point) A capacitor is connected to a battery until charged and remains connected. What happens when you fill the space between the plates with glass, a dielectric?

A. The electric field between the plates decreases.
B. There can be no field inside the dielectric.
C. The voltage across the plates increases.
[✓ D.] The magnitude of the charges on the plates increase.

1m) (3 points) Consider a parallel-plate capacitor with plate-area A and (initial) plate-separation d hooked to a relaxed insulating spring with stiffness k, as pictured below. Charges Q and $-Q$ are placed on the plates of the capacitor and the plate attached to the spring moves slightly toward the other plate. What is the displacement of the plate? (Assume no friction between the capacitor and the surface.)

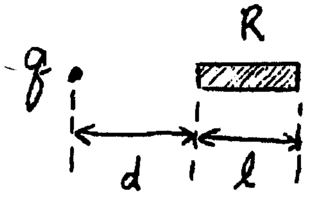


new equilibrium ($F_{net} = 0$)

$$kx = QE \qquad \Rightarrow \quad x = \frac{QE}{k} = \frac{Q^2}{kA\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

1n) (4 points) Consider a resistor of length ℓ and resistance R in the presence of a charge $q > 0$, as pictured below, with the charge along the axis of the resistor and at a distance d away from the resistor. Neither the charge nor the resistor are moving. What is the voltage (or change in electric potential) across the resistor? Why? What then must be the current in the resistor, and what is its direction?

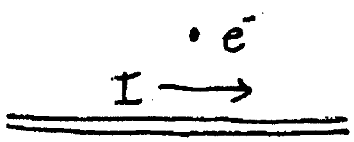


$\Delta V = 0$ since resistor is a conductor in electrostatic equil. (it has been polarized)

$I = \frac{\Delta V}{R} = 0$ (charge moved initially when resistor was polarized – rightward – but now, no current)

no direction

1o) (2 points) If you place an electron next to a current-carrying wire, as shown below, in which direction does the electric force on the electron point? And in which direction does the magnetic force on the electron point? Explain (very briefly).



$\vec{F}_E = \vec{0}$ No net charge nearby (wire is neutral)

Unless e^- induces polarization in wire
⇒ attractive force toward wire

$\vec{F}_B = -e\vec{v}\times\vec{B} = \vec{0}$

since $\vec{v} = \vec{0}$

1p) (1 point) In which configuration will two 60-watt light bulbs shine brighter, when connected in series or in parallel?

Wattage is a bad label for bulbs, in general, since it can change, depending on the circumstances.

A. in series
[✓ B. in parallel]
C. it makes no difference – they are 60-watt light bulbs

1q) (1 point) You have three resistors of equal resistance, but you need a lower resistance. Can you combine the three resistors to get a lower resistance?

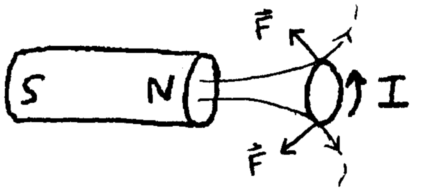
A. No, you can only combine them to get more resistance.
B. Yes, you can connect them in series.
[✓ C. Yes, you can connect them in parallel.]

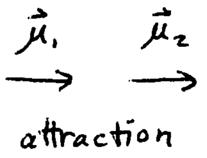
1r) (1 point) A proton and an electron move in circular orbits within a uniform magnetic field. If both particles move with the same speed, which will have the larger orbit?

[✓ A. The proton has the larger orbit.] Same magnitude of centripetal force, larger inertia.
B. The electron has the larger orbit.
C. Since both have the same charge, they move in exactly the same orbit.
D. Their orbits have the same radius, but the directions are reversed.

1s) (1 point) Consider a cylindrical magnet, shown below, with a magnetic field that is symmetric about its axis. Suppose a planar circular loop of current is centered on the magnet's axis with its plane perpendicular to the axis. The loop is positioned near the north pole of the magnet with the current $I > 0$ in the loop directed as shown below. What can you say about the net magnetic force on the loop of current?

A. The net force is perpendicular to the magnetic field at the center of the loop.
[✓ B. The net force is directed toward the magnet.]
C. The net force is directed away from the magnet.
D. The net force is zero.



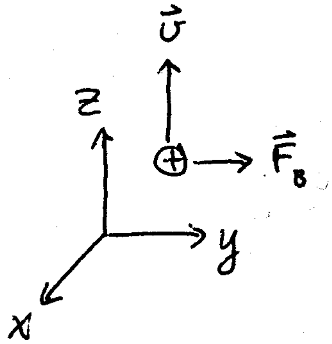


1t) (1 point) Consider the same magnet and loop of current pictured above. What can you say about the net magnetic torque on the loop of current?

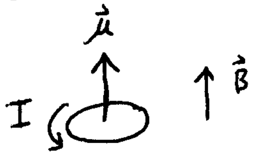
A. The net torque is perpendicular to the magnetic field at the center of the loop.
B. The net torque is directed toward the magnet.
C. The net torque is directed away from the magnet.
[✓ D. The net torque is zero.] Symmetric forces

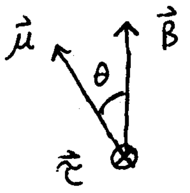
1u) (3 points) A positively charged particle moves in the positive z-direction. The magnetic force on the particle is in the positive y-direction. What can you conclude about the...

- x-component of the magnetic field at the particles position?
  - [✓ i.] $B_x > 0$
  - ii. $B_x = 0$
  - iii. $B_x < 0$
  - iv. not enough information given to decide
- y-component of the magnetic field at the particles position?
  - i. $B_y > 0$
  - [✓ ii.] $B_y = 0$
  - iii. $B_y < 0$
  - iv. not enough information given to decide
- z-component of the magnetic field at the particles position?
  - i. $B_z > 0$
  - ii. $B_z = 0$
  - iii. $B_z < 0$
  - [✓ iv.] not enough information given to decide



1v) (5 points) Suppose a planar loop of current I with loop-area A is in a uniform magnetic field of magnitude B, initially at rest with its magnetic moment aligned with the field. The dipole is able to rotate about an axis that is perpendicular to the magnetic field, and it has a moment of inertia Γ about this axis. If the dipole is perturbed a small angle away from being aligned and is then free to oscillate (with no damping), what is the period of these oscillations?





$$\vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin\theta \, \hat{n}$$

$$\tau = -\mu B \sin\theta \approx -\mu B \theta = \Gamma \alpha$$

$$\Rightarrow \alpha = -\frac{\mu B}{\Gamma} \theta$$

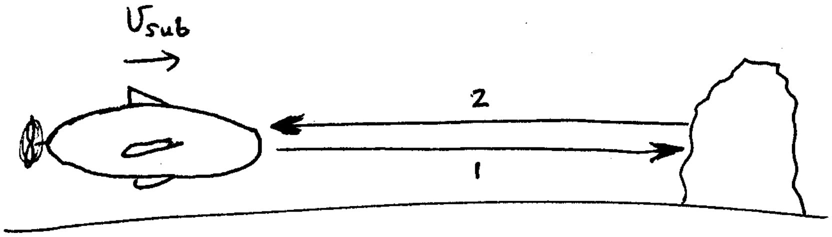
$$\Rightarrow \omega_0 = \sqrt{\frac{\mu B}{\Gamma}}$$

$$\Rightarrow T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{\Gamma}{\mu B}} = 2\pi \sqrt{\frac{\Gamma}{I A B}}$$

2. *Submarine race.*

(14 points) A submarine pilot is in a race and must navigate around obstacles. She uses an intermittent series of short sound pulses for sonar (sound navigation and ranging). That is, she uses these sound pulses to detect the presence of and distance to stationary obstacles and also to calculate her speed in the water. Each sound pulse, or "ping", has a frequency of 350.00 Hz, and her digital sonar equipment calculates the speed of sound in the water around her to be 1550 m/s. If a pulse is emitted and returns 27.2 ms later with a frequency of 351.00 Hz, how far away is the obstacle and how fast is the submarine moving toward the obstacle at this moment (assuming the submarine is headed straight for the obstacle)?

$$2d = v_m \Delta t \quad (\text{* assuming } v_{sub} << v_m)$$



$$\Rightarrow d = \frac{1}{2} v_m \Delta t$$
$$= \frac{1}{2}(1550 \tfrac{m}{s})(27.2 \text{ ms})$$
$$= 21.1 \text{ m}$$

$$1: \quad f_o = \left(\frac{v_m + v_o}{v_m - v_s}\right) f_s = \left(\frac{v_m}{v_m - v_{sub}}\right) f .$$

$$2: \quad f_o' = \left(\frac{v_m + v_o}{v_m - v_s}\right) f_s' = \left(\frac{v_m + v_{sub}}{v_m}\right) f_o = \left(\frac{v_m + v_{sub}}{\cancel{v_m}}\right)\left(\frac{\cancel{v_m}}{v_m - v_{sub}}\right) f$$

$$f_o'(v_m - v_{sub}) = (v_m + v_{sub}) f$$

$$v_m (f_o' - f) = v_{sub}(f_o' + f)$$

$$v_{sub} = \left(\frac{f_o' - f}{f_o' + f}\right) v_m = \left(\frac{351.00 - 350.00}{351.00 + 350.00}\right)(1550 \tfrac{m}{s})$$

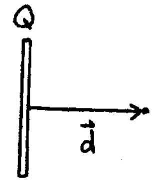
$$= 2.21 \text{ m/s}$$

$$(v_{sub} << v_m \ \checkmark)$$

3. *Hexagonal geometries.*

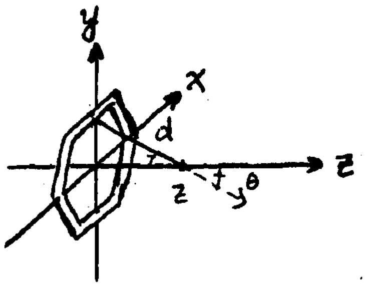
In class we examined a finite line of charge of length ℓ with uniform linear charge density and total charge Q. We found that the electric field due to this line charge at a point a (perpendicular) displacement **d** away from the center of the line is

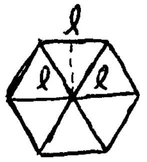
$$\mathbf{E} = \frac{kQ}{d}\frac{1}{\sqrt{d^2 + (\ell/2)^2}}\hat{\mathbf{d}}.$$

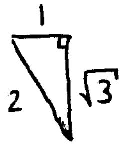


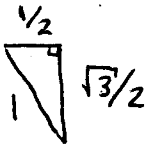
Use this formula to solve the following problems.

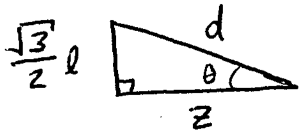
3a) (10 points) Consider a hexagonal loop of charge with uniform linear charge density and total charge q, pictured below. The loop is in the xy-plane, centered at the origin, and each side of the loop has length ℓ. Calculate the electric field at any point on the z-axis due to this charge.











$$d = \sqrt{z^2 + (\sqrt{3}\,\ell/2)^2}$$
$$= \sqrt{z^2 + 3(\ell/2)^2}$$

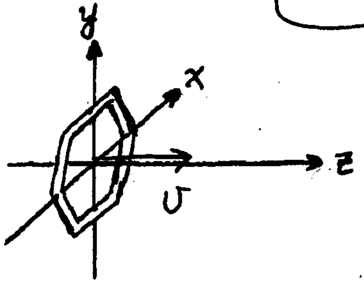
symmetry ⇒ cancellation w/ only z-component remaining

$$\vec{E} = 6\,E_z'\,\hat{z}$$
$$= 6\,E'\cos\theta\,\hat{z} = 6\,E'\,\frac{z}{d}\,\hat{z}$$
$$= 6\left(\frac{k\,q/6}{d}\,\frac{1}{\sqrt{d^2 + (\ell/2)^2}}\right)\frac{z}{d}\,\hat{z}$$
$$= \frac{k\,q\,z}{d^2}\,\frac{1}{\sqrt{d^2 + (\ell/2)^2}}\,\hat{z}$$
$$= \frac{k\,q\,z}{(z^2 + 3(\ell/2)^2)}\,\frac{1}{\sqrt{z^2 + 4(\ell/2)^2}}\,\hat{z}$$

3b) (7 points) Suppose a particle with the same charge q as the loop and mass m is fired through the middle of the loop. When it is at the center of the loop, the particle has a speed v in the z-direction.

- At what speed will the particle be traveling when it is very far away, moving with an essentially constant velocity?
- Is this eventual speed greater or less than the original speed v?

Your answer will involve an integral. You do not need to solve the integral, but express it as fully as you can and pull the constants out in front of the integral. Assume there are no dissipative forces.



Conservation of energy

K.E. + P.E.

$$\frac{1}{2}mv^2 + qV = \frac{1}{2}mv'^2 + 0$$

$$v' = \sqrt{v^2 + \frac{2q}{m}V}$$

$$V = -\int_{\infty}^{\text{origin}} \vec{E}\cdot d\vec{\ell} = -\int_{\infty}^{0} E_z\,dz \quad (\text{along } z\text{-axis})$$

$$= \int_{0}^{\infty} E_z\,dz = kq\int_{0}^{\infty} \frac{z\,dz}{(z^2 + 3(\ell/2)^2)\sqrt{z^2 + 4(\ell/2)^2}}$$

loop q, particle q ⇒ Same/like charges repel

⇒ $v' > v$

4. *Double-sphere capacitor.*

Consider the circuit pictured below. A battery of emf $\mathcal{E}$ is hooked to two spherical capacitors in the manner shown. The top capacitor contains a dielectric of constant K_1 and has radii R_1, R_2, and R_3 for each boundary, from smallest to largest. The bottom capacitor contains a dielectric of constant K_2 and has radii R_4, R_5, and R_6, smallest to largest.

For the battery to reach the inner conductors, each capacitor has a small hole drilled in the outer conducting shell so a wire can be fed in to touch the inner conductor without touching the outer conductor. Assume that this does not measurably affect the symmetry within the capacitors.

4a) (5 points) Where will charge accumulate in this circuit, and what will be the signs of those charges? Label these charges.

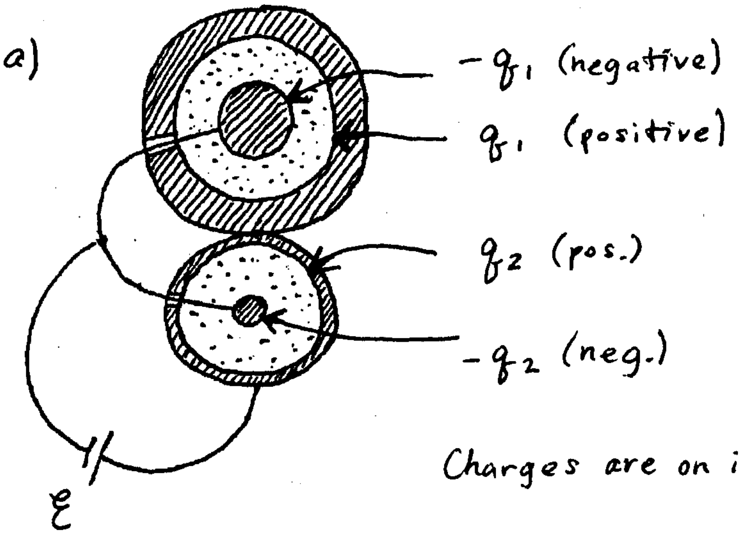
4b) (5 points) In terms of these labeled charges and other parameters, write expressions for the electric field inside these capacitors.

4c) (10 points) What are the magnitudes of the accumulated charges?

4d) (5 points) What is the effective capacitance of this arrangement?

(capacitors cross-sectioned)

a)



Charges are on inside surfaces.

b) top capacitor: $\vec{E}_1 = \frac{1}{K_1}\frac{-q_1}{4\pi r^2 \varepsilon_0}\hat{r} = -\frac{kq_1}{K_1 r^2}\hat{r}$

bottom capacitor: $\vec{E}_2 = \frac{1}{K_2}\frac{-q_2}{4\pi r^2 \varepsilon_0}\hat{r} = -\frac{kq_2}{K_2 r^2}\hat{r}$

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c) $q_1 = C_1 V_1 = C_1 \mathcal{E}$

$$C_1 = \frac{q_1}{V_1}$$

$$V_1 = -\int_{R_1}^{R_2} E_{1r}\, dr = +\frac{k q_1}{K_1} \int_{R_1}^{R_2} \frac{dr}{r^2}$$

$$= \frac{k q_1}{K_1} \left[-\frac{1}{r} \right]_{R_1}^{R_2} = \frac{k q_1}{K_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$C_1 = \frac{q_1}{V_1} = \frac{K_1}{k \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}$$

$$q_1 = C_1 \mathcal{E} = \frac{K_1 \mathcal{E}}{k \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}$$

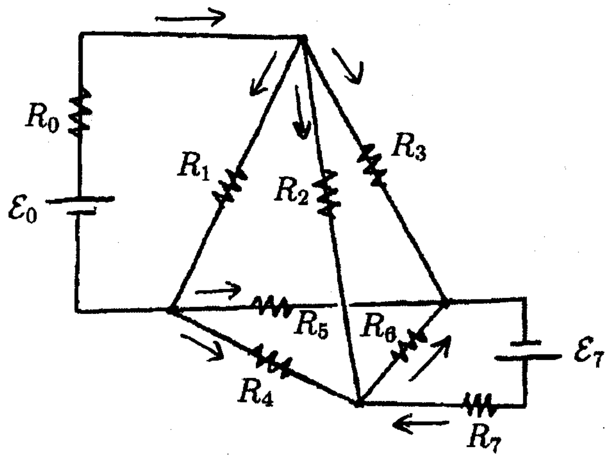
$$q_2 = C_2 \mathcal{E} = \frac{K_2 \mathcal{E}}{k \left(\frac{1}{R_4} - \frac{1}{R_5} \right)}$$

d) C_{eq}? parallel! (same voltage, $q_{tot} = q_1 + q_2$)

$$\Rightarrow C_{eq} = C_1 + C_2 = \frac{K_1}{k \left(\frac{1}{R_1} - \frac{1}{R_2} \right)} + \frac{K_2}{k \left(\frac{1}{R_4} - \frac{1}{R_5} \right)}$$

5. *Resistor circuit.*

(15 points) Assuming the resistances and emf's in the circuit shown below are known, how many independent equations are needed to solve for all the currents? Create enough equations to be able to solve the unknown currents in the circuit. (Do not solve the equations, just set them up.) The circuit includes a 3D triangular pyramid (tetrahedron). The small apparent break in the rear wire is to help you visualize which segment is behind the others – there are no breaks in the circuit.



8 branches (8 currents, 8 unknowns)

⇒ need 8 independent equations

(There are ~6 simple loops, but only need 5 due to redundancy. There are 4 nodes, but only need 3 due to redundancy.)

<u>K1</u>

$$\mathcal{E}_0 - R_0 I_0 - R_1 I_1 = 0$$

(back) $$R_1 I_1 - R_3 I_3 + R_5 I_5 = 0$$

(left) $$R_1 I_1 - R_2 I_2 + R_4 I_4 = 0$$

(right) $$R_2 I_2 - R_3 I_3 + R_6 I_6 = 0$$

(bottom) $$-R_5 I_5 + R_6 I_6 + R_4 I_4 = 0$$

$$\mathcal{E}_7 - R_7 I_7 - R_6 I_6 = 0$$

<u>K2</u>

$$I_0 = I_1 + I_2 + I_3$$

$$I_1 = I_0 + I_4 + I_5$$

$$I_7 = I_3 + I_5 + I_6$$

$$I_6 = I_2 + I_4 + I_7$$

overkill.
10 equations

6. *RC circuit.*

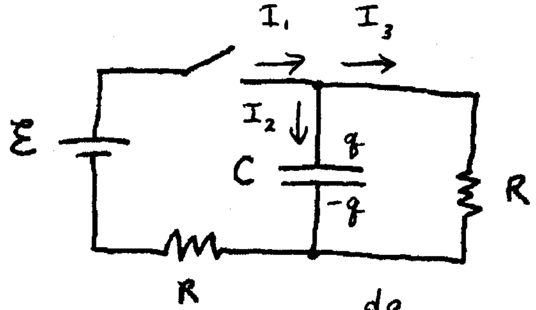
Consider the circuit pictured below. Initially, before the switch is closed, the capacitor is completely neutral and uncharged.

6a) (4 points) Immediately after the switch is closed, what will the currents and charge on the capacitor be?

6b) (4 points) A long time after the switch is closed, what will the currents and charge on the capacitor be?

6c) (12 points) For any time after the switch is closed, solve for the current to the capacitor and the charge on the capacitor as functions of time.

Hint: You may need to take a derivative of equations that you come up with.



$$I_2 = \frac{dq}{dt}$$

a) capacitor's like a short

$q = 0$

$I_1 = I_2 = \frac{\mathcal{E}}{R} \qquad I_3 = 0$

b) capacitor's like a break

$I_1 = I_3 = \frac{\mathcal{E}}{2R} \qquad I_2 = 0$

$q = CV = C I_3 R = \frac{C\mathcal{E}}{2}$

c)

$$\frac{d}{dt}\left[\begin{array}{l} \mathcal{E} - \frac{q}{C} - R I_1 = 0 \\ \frac{q}{C} - R I_3 = 0 \\ I_1 = I_2 + I_3 \end{array}\right] = \left[\begin{array}{l} -\frac{1}{C} I_2 - R\frac{dI_1}{dt} = 0 \\ \frac{1}{C} I_2 - R\frac{dI_3}{dt} = 0 \\ \frac{dI_1}{dt} = \frac{dI_2}{dt} + \frac{dI_3}{dt} \end{array}\right] \begin{array}{l} \rightarrow \underbrace{\frac{dI_1}{dt} = -\frac{1}{RC} I_2} \\ \nearrow \frac{1}{RC} I_2 = \frac{dI_3}{dt} = \frac{dI_1}{dt} - \frac{dI_2}{dt} \\ \frac{1}{RC} I_2 = -\frac{1}{RC} I_2 - \frac{dI_2}{dt} \end{array}$$

$$\frac{dI_2}{dt} = -\frac{2}{RC} I_2 \qquad \int_{\mathcal{E}/R}^{I_2} \frac{dI_2'}{I_2'} = \int_0^t -\frac{2}{RC}\,dt' \qquad \ln\left(\frac{I_2}{\mathcal{E}/R}\right) = -\frac{2}{RC} t$$

$$I_2 = \frac{\mathcal{E}}{R} e^{-2t/RC} \qquad q = \int_0^q dq' = \int_0^t I_2\,dt' = \frac{\mathcal{E}}{R}\left(-\frac{RC}{2}\right)\left[e^{-2t/RC} - 1\right]$$

$$= \frac{C\mathcal{E}}{2}\left(1 - e^{-2t/RC}\right)$$