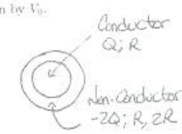
A conducting sphere of radius R carries a charge Q. It is surrounded by a non-conducting blanket that extends from R to $2R$ and carries a charge $-2Q$.

- a) (10 pts) Find the net charge inside a concentric sphere of radius r for all values of r.
- b) (5 pts) Find the electric field (vector) as a function of distance from the center of the distribution (r).
- c) (10 pts) Find the electric potential as a function of distance from the center of the distribution (r) if the potential in the center of the distribution is given by V_0.



a) $(r<R)$ $\boxed{q_{in}(r) = 0}$

$(R<r<2R)$ $q_{in}(r) = Q + (-2Q)\,\dfrac{r^3 - R^3}{(2R)^3 - R^3}$

$$q_{in}(r) = Q\left[1 - 2\,\frac{r^3 - R^3}{7R^3}\right]$$

$$\boxed{q_{in}(r) = \frac{Q}{7}\left(9 - 2\left(\tfrac{r}{R}\right)^3\right)}$$

$(2R<r)$ $\boxed{q_{in}(r) = -Q}$

b) Spherical Symmetry, So $\vec{E} = \dfrac{q_{in}(r)}{4\pi\epsilon_0 r^2}\,\hat{r}$

$$\boxed{\vec{E} = \begin{cases} 0 & (r<R) \\ \dfrac{Q}{4\pi\epsilon_0 R^2}\,\dfrac{1}{7}\left[9\left(\frac{R}{r}\right)^2 - 2\left(\frac{r}{R}\right)\right]\hat{r} & (R<r<2R) \\ \dfrac{-Q}{4\pi\epsilon_0 r^2}\,\hat{r} & (2R<r) \end{cases}}$$

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c) $\Delta V(0,r) = V(r) - V(0) = V(r) - V_0$ So...

$V(r) = V_0 + \Delta V(0,r)$

$V(r) = V_0 - \int_0^r \vec{E}(r)\cdot d\vec{r}$ $\quad\supset$ Spherical Symmetry

$V(r) = V_0 - \int_0^r E(r)\,dr$

$(r<R)$ $V(r) = V_0 - \int_0^r (0)\,dr$

$\boxed{V(r) = V_0}$

$(R<r<2R)$ $V(r) = V_0 - \int_0^R (0)\,dr - \int_R^r \dfrac{Q}{28\pi\epsilon_0 R}\left(9\frac{R}{r^2} - 2\frac{r}{R^2}\right)dr$

$$V(r) = V_0 + \frac{Q}{28\pi\epsilon_0 R}\left(9\,\tfrac{R}{r} + \left(\tfrac{r}{R}\right)^2\right)\Big|_R^r$$

$$\boxed{V(r) = V_0 + \frac{Q}{28\pi\epsilon_0 R}\left(9\tfrac{R}{r} + \left(\tfrac{r}{R}\right)^2 - 10\right)}$$

$(2R<r)$ $V(r) = V_0 - \int_0^R (0)\,dr - \int_R^{2R} \dfrac{Q}{28\pi\epsilon_0 R}\left(9\frac{R}{r^2} - 2\frac{r}{R^2}\right)dr - \int_{2R}^r \dfrac{-Q}{4\pi\epsilon_0 r^2}\,dr$

$$V(r) = V_0 - \frac{3Q}{56\pi\epsilon_0 R} - \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{2R}\right)$$

$$\boxed{V(r) = V_0 + \frac{Q}{14\pi\epsilon_0 R} - \frac{Q}{4\pi\epsilon_0 r}}$$