



- a) (10 pts) A thin nonconducting rod is bent to form a circular arc of radius R that subtends an angle $2\theta_0$ as shown in the diagram on the left. If that rod carries a charge Q (uniformly distributed), show that the magnitude of the electric field at point A will be $E = \frac{Q}{4\pi\epsilon_0 R^2 \theta_0}$. Show your work!
- b) (10 pts) Now consider the wedge in the center diagram. If a charge q is uniformly distributed over the region that extends from R_1 to R_2 (subtending an angle $2\theta_0$), what is the magnitude of the electric field at point B [See the hint in the right diagram]
- c) (5 pts) Suppose a small non-uniform but spherically-symmetric charge distribution of total charge Q is placed at point B . What is the magnitude of the force that the charge Q will exert on the wedge?

Handwritten calculations on the right side of the page:

$$dq = \lambda dl = \lambda R d\theta$$

$$\lambda = \frac{Q}{2\pi R \theta_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 R^2 \theta_0}$$

$$A = \frac{1}{2} r^2 \theta$$

3) $d\vec{E} = \frac{k dq}{r^2} \hat{r}$ $r = R$

$$d\vec{E} = \frac{k \lambda R d\theta}{R^2} (\sin\theta \hat{i} - \cos\theta \hat{j}) \quad R = \sqrt{r^2}$$

$$d\vec{E} = \frac{k \lambda d\theta}{R} (\sin\theta \hat{i} - \cos\theta \hat{j})$$

$$\vec{E} = \int d\vec{E} = \frac{k \lambda}{R} \int_0^{\theta_0} (\sin\theta \hat{i} - \cos\theta \hat{j}) d\theta$$

$$\vec{E} = \frac{k \lambda}{R} (\cos\theta_0 - 1) \hat{i} - \frac{k \lambda}{R} \sin\theta_0 \hat{j}$$

$|\vec{E}| = \frac{k \lambda \sin\theta_0}{R}$

$$S = \frac{Q}{2(120)(4\pi \cdot 10^{-12})}$$

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$$dA = r(\theta_0) d\theta$$

$$dA = 2R \sin\theta d\theta$$

EXTRA PAGE

b) Build the wedge up of infinitesimal arcs

$$d\vec{E} = \frac{k dq}{r^2} \hat{r} = \frac{k \lambda r d\theta}{r^2} \hat{r} \quad dq = \lambda r d\theta = \frac{Q}{2\pi R} r d\theta$$

$$\vec{E} = \int d\vec{E} = \frac{k \lambda}{R} \int_0^{\theta_0} \frac{r}{r^2} \hat{r} d\theta$$

$|\vec{E}| = \frac{k \lambda \sin\theta_0}{R \cos\theta_0} \ln\left(\frac{R_2}{R_1}\right)$

c) By Newton's third law, the force exerted on the charge Q is equal in magnitude and opposite in direction to the force exerted on the wedge.

$$|\vec{F}| = |Q\vec{E}|$$

$$|\vec{F}| = \frac{Q^2}{4\pi\epsilon_0 R^2 \theta_0}$$