MT1 Physics 1B W19

Full Name (Printed) [redacted]

Full Name (Signature) [redacted]

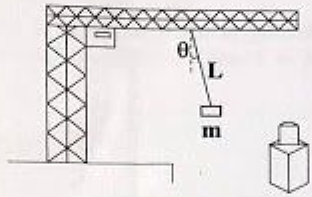
Student ID Number [redacted]

Seat Number 首席

Problem	Grade
1	12 /30
2	15 /30
3	18 /30
Total	45 /90

45

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



A crane is used to move heavy cargo containers from a docked ship to the shore. During one such offload, the transport suddenly locks-up, inducing a dangerous sway that turns the crate into a large pendulum of mass m and length L. A nearby safety-officer notes that it takes twenty complete cycles for the oscillations to lose about half of their amplitude.

Assume that the damping torque that acts on the crate can be modeled as $\vec{\tau}_{damp} = -\Gamma \frac{d\vec{\theta}}{dt}$, where Γ is, for now, unknown, and answer the following questions...

- 1a) (10 points) Use Newton's Laws in one form or another, or work and energy to obtain the differential equation that describes the motion of the system (in terms of θ, assuming small angles) and write the solution for that equation. You may use m, and L and Γ in your answers.

$\tau = I\alpha$

$\tau = r \times F$

$L\sin\theta \quad F = mg$

$-\Gamma\dot{\theta} - mgL\sin\theta = mL^2\ddot{\theta}$

$$\boxed{\ddot{\theta} + \frac{\Gamma}{mL^2}\dot{\theta} + \frac{g}{L}\theta = 0}$$

$$\theta = \theta_{max} e^{\frac{-\Gamma t}{2mL^2}} \cos(\omega t + \phi)$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

$$= \sqrt{\frac{g}{L} - \left(\frac{\Gamma}{2mL}\right)^2}$$

4

- 1b) (15 points) Find the angular frequency at which the pendulum is oscillating. Your answer should be in terms of the given information, you should not have Γ in the answer.

$$\omega_{damp} = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

$$\omega_{damp} = \sqrt{\omega_0^2 - \left(\frac{\Gamma}{2mL^2}\right)^2}$$ +2

$A(0) = A_0$

$\frac{1}{2}A_0 = A_0 e^{-\frac{bt}{2m}}$

$\frac{1}{2} = e^{-\frac{bt}{2m}}$

$2 = e^{\frac{bt}{2m}}$

$2 = e^{\frac{\Gamma}{2mL^2}t}$

NT

$\ln 2 = \frac{\Gamma}{2mL^2} NT$

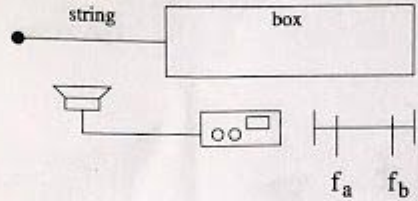
$\frac{\ln 2}{T(20)} = \frac{\Gamma}{2mL^2}$

+6

$$\boxed{\omega_{damp} = \sqrt{\omega_0^2 - \frac{\ln 2}{20}T}}$$

- 1c) (5 points) Find the damping coefficient (Γ).

$\Gamma = \frac{m}{mL^2}$



2) The visible end of a taut string (mass m, tension T) is fixed. The opposite end (and much of the string's length) is obscured by a box which is open on both ends. The speed of sound in air is v_{snd}.

A signal generator sits nearby. As the signal generator is tuned through the range f_{lo} to f_{hi} the string is observed to vibrate at two and only two frequencies: f_a and f_b.

- 2a) (10 points) How might one use this information to determine whether the obscured end of the string is fixed or loose (without peeking)? Provide as much detail as you can.

The f_a is the fundamental harmonic freq. If you divide f_b by f_a, and you get a whole number, then would mean that those harmonics are next to each other and thus should be a fixed end at the right end of the box. If you get a decimal, then those harmonics are 2 apart, hence making the end of the box a loose end in this case.

+8

- 2b) (5 points) **For the rest of the problem, we'll suppose the obscured end is loose.** In terms of the given information, which harmonic does f_b correspond to? How many intermediate nodes (that is, nodes between the ends) are on the string when it vibrates in this mode?

f_b should correspond to the 3rd harmonic. +1

This should have 2 intermediate nodes.

- 2c) (5 points) How long is the string?

$$f_N = \frac{v_x}{4L}$$

$$f_a = \sqrt{\frac{T}{\mu}} / 4L \quad +1$$

$$f_a^2 = \frac{TL}{m\,16L^2}$$

$$f_a^2 = \frac{T}{16mL}$$

$$L = \frac{T}{16m f_a^2} \quad +1$$

- 2d) (5 points) Suppose that the frequencies f_a and f_b also set the box into resonance. Will there be any other frequencies within the tunable range that set the box into resonance? Explain.

Since this is a mixed system, any

$+2$ $f_N = (2n+1)\frac{v_x}{4L}$ should set the box into resonance.

Hence, $f_N = (2n+1) f_a$ should do the trick.

- 2e) (5 points) If the box resonates at f_a and f_b, what values might its length have?

$V_x = \sqrt{\frac{B}{\rho}}$ ← whatever these are

$$\frac{\lambda}{4} = \frac{\sqrt{\frac{B}{\rho}}}{f_a} \quad +1$$

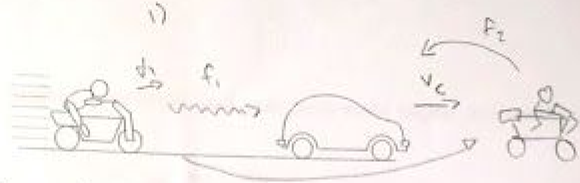
$$f_a = \frac{V_x}{4L}$$

$$f_a = \frac{\sqrt{\frac{B}{\rho}}}{4L}$$

$$f_a^2 = \frac{B}{16\rho L^2}$$

$$+1 \quad L^2 = \frac{B}{16\rho f_a^2}$$

$$L = \sqrt{\frac{B}{16\rho f_a^2}}$$



3) Driving along the freeway with a speed v_c, you are suddenly overtaken by a much faster motorcycle (trying desperately, no doubt, to flee pursuing raptors, but I digress). As the motorcycle approaches, you observe the dominant component of the sound emanating from its screaming occupant to have a frequency f_1. Once the motorcycle has passed (but before the raptors have caught up to you), that dominant component of the screams is observed to shift to a frequency f_2.

- 3a) (15 points) Find the speed at which the motorcycle is traveling.

$$\frac{f_1}{f_{src}} = \frac{V_{snd} - V_c}{V_{snd} - V_{bike}} \qquad \frac{f_2}{f_{src}} = \frac{V_{snd} + V_c}{V_{snd} + V_{bike}}$$

$$\frac{f_1(V_{snd} - V_{bike})}{V_{snd} - V_c} = f_{src}$$

$$\frac{f_2}{\frac{f_1(V_{snd} - V_{bike})}{V_{snd} - V_c}} = \frac{V_{snd} + V_c}{V_{snd} + V_{bike}}$$

$$\frac{(V_{snd} - V_c) f_2}{f_1(V_{snd} - V_{bike})} = \frac{V_{snd} + V_c}{V_{snd} + V_{bike}}$$

$$(V_{snd} - V_c) f_2 (V_{snd} + V_{bike}) = (V_{snd} + V_c) f_1 (V_{snd} - V_{bike})$$

$$(V_{snd} - V_c) f_2 (V_{snd}) + (V_{snd} - V_c)(f_2) V_{bike} = (V_{snd} + V_c)(f_1)(V_{snd}) - (V_{snd} + V_c)(f_1)(V_{bike})$$

$$f_2 V_{snd}^2 - V_c V_{snd} f_2 + \ldots = V_{snd}^2 f_1 - V_c f_1 V_{snd}$$

$$f_2 V_{snd}^2 - V_c V_{snd} f_2 - V_{snd}^2 f_1 - V_c f_1 V_{snd} = -((V_{snd} + V_c)(f_1) + (V_{snd} - V_c)(f_2)) V_{bike}$$

$$\boxed{V_{bike} = \frac{-f_2 V_{snd}^2 + V_c V_{snd} f_2 + V_{snd}^2 f_1 + V_c f_1 V_{snd}}{(V_{snd} + V_c)(f_1) + (V_{snd} - V_c)(f_2)}}$$

1) $\frac{f_{obs}}{f_{src}} = \frac{V_{snd} - V_c}{V_{snd} - V_{bike}}$ 2) $\frac{f_{obs}}{f_{src}} = \frac{V_{snd} + V_c}{V_{snd} + V_{bike}}$

$\frac{f_{obs}}{f_1} = \frac{V_{snd} - V_c}{V_{snd} - V_{bike}}$ $\frac{f_{obs}}{f_2} = \frac{V_{snd} + V_c}{V_{snd} + V_{bike}}$

$f_{obs}(V_{snd} - V_{bike}) = (V_{snd} - V_c) f_1$

$f_{obs}(V_{snd}) - f_{obs}(V_{bike}) = (V_{snd} - V_c) f_1$

$\frac{f_{obs}(V_{snd}) - (V_{snd} - V_c) f_1}{f_{obs}} = V_{bike}$

$$\frac{f_{obs}}{f_2} = \frac{V_{snd} + V_c}{V_{snd} + V_{snd} - \frac{V_{snd} f_1}{f_{obs}} + \frac{V_c f_1}{f_{obs}}}$$

$2V_{snd} f_{obs} - V_{snd} f_1 + V_c f_1 = (V_{snd} + V_c) f_2$

$2V_{snd} f_{obs} = (V_{snd} + V_c) f_1 + V_{snd} f_1 - V_c f_1$

$f_{obs} = \frac{(V_{snd} + V_c) f_1 + V_{snd} f_1 - V_c f_1}{2V_{snd}}$

$V_{snd} f_1 - V_c f_1 - (V_{snd} - V_c) f_1$ $(V_{snd} + V_c) f_1 + V_{snd} f_1 - V_c f_1$ $2V_{snd}$ $= V_{motorcycle}$

+15

- 3b) (10 points) What frequency would that dominant component appear to have if you could match the motorcycle's speed?

$$\frac{f_{obs}}{f_{src}} = \frac{V_{snd} - V_{same}}{V_{snd} - V_{same}}$$

$$f_{obs} = f_{src}$$

- 3c) (5 points) Evaluate your answers to parts a and b in the limit $f_2 \to f_1$ and explain the results.

As $f_2 \to f_1$ ..

$$V_{bike} = \frac{\cancel{-f_1 V_{snd}^2} + V_c V_{snd} f_1 + \cancel{V_{snd}^2 f_1} + V_c f_1 V_{snd}}{(V_{snd} + V_c) f_1 + (V_{snd} - V_c) f_2}$$

$$V_{bike} = \frac{2 V_c V_{snd}}{2 V_{snd}}$$

$$V_{bike} = V_c$$

+3

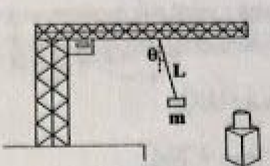
As $f_2 \to f_1$, the speed should become the same as well so then the doppler shift does not happen.

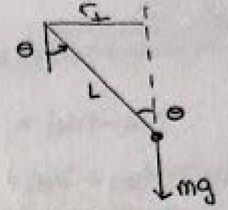
F

$\tau_g = -mgL\sin\theta$

$\tau_d = -\Gamma\frac{d\theta}{dt}$

Small angles — $\sin\theta \approx \theta$





A crane is used to move heavy cargo containers from a docked ship to the shore. During one such offload, the transport suddenly locks-up, inducing a dangerous sway that turns the crate into a large pendulum of mass m and length L. A nearby safety-officer notes that it takes twenty complete cycles for the oscillations to lose about half of their amplitude.

Assume that the damping torque that acts on the crate can be modeled as $\vec{\tau}_{damp} = -\Gamma\frac{d\theta}{dt}$, where Γ is, for now, unknown, and answer the following questions...

- 1a) (10 points) Use Newton's Laws in one form or another, or work and energy to obtain the differential equation that describes the motion of the system (in terms of θ, assuming small angles) and write the solution for that equation. You may use m, and L and Γ in your answers.

$\Sigma\vec{\tau} = I\vec{\alpha}$

$-mgL\sin\theta - \Gamma\frac{d\theta}{dt} = mL^2\frac{d^2\theta}{dt^2}$

$\frac{d^2\theta}{dt^2} + \frac{\Gamma}{mL^2}\frac{d\theta}{dt} + \frac{g}{L}\theta = 0$

Compare to...

$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{K}{m}x = 0$

$x = A_0 e^{-\frac{bt}{2m}}\cos(\omega t + \phi)$

$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$

$$\boxed{\begin{gathered}\frac{d^2\theta}{dt^2} + \frac{\Gamma}{mL^2}\frac{d\theta}{dt} + \frac{g}{L}\theta = 0 \\ \theta = \theta_{max,o}\, e^{-\frac{\Gamma t}{2mL^2}}\cos(\omega t + \phi) \\ \omega = \sqrt{g/L - \left(\frac{\Gamma}{2mL^2}\right)^2}\end{gathered}}$$

$\theta_{max,o}$ and ϕ would be determined by initial conditions (not given)

$$\theta_{max}(t) = \theta_{max,0}\, e^{-\frac{\Gamma t}{2mL^2}} \Rightarrow \frac{\theta_{2max}}{\theta_{1max}} = e^{-\frac{\Gamma(t_2-t_1)}{2mL^2}}$$

$$\begin{cases} t_2 - t_1 = NT \quad (N=20) \\ T = \dfrac{2\pi}{\sqrt{\omega_0^2 - \left(\frac{\Gamma}{2mL}\right)^2}} \\ \dfrac{\theta_{2max}}{\theta_{1max}} = 1/2 \end{cases}$$

- 1b) (15 points) Find the angular frequency at which the pendulum is oscillating. Your answer should be in terms of the given information, you should not have Γ in the answer.

$$\ln(1/2) = -2\pi N \frac{\Gamma}{2mL^2} \frac{1}{\sqrt{\omega_0^2 - (\Gamma/2mL)^2}}$$

$$\left(\frac{2\pi N}{\ln(2)}\right)^2 = \left(\frac{2mL^2\omega_0}{\Gamma}\right)^2 - 1$$

(N=20)

$$\left(\frac{\Gamma}{2mL^2\omega_0}\right)^2 = \frac{1}{1 + \left(\frac{40\pi}{\ln(2)}\right)^2}$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{\Gamma}{2mL^2}\right)^2}$$

$$\omega = \omega_0 \sqrt{1 - \left(\frac{\Gamma}{2mL^2\omega_0}\right)^2}$$

$$\omega_0 = \sqrt{g/L}$$

$$\boxed{\omega = \sqrt{\frac{g}{L}}\, \frac{40\pi/\ln(2)}{\sqrt{1 + \left(\frac{40\pi}{\ln(2)}\right)^2}}}$$

- 1c) (5 points) Find the damping coefficient (Γ).

$$\Gamma = \frac{2mL^2\omega_0}{\sqrt{1 + \left(\frac{40\pi}{\ln(2)}\right)^2}}$$ From part b, $\omega_0 = \sqrt{g/L}$

$$\boxed{\Gamma = \frac{2mL\sqrt{gL}}{\sqrt{1 + \left(\frac{40\pi}{\ln(2)}\right)^2}}}$$

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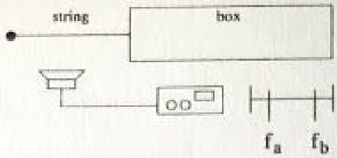
$\cos(\omega t + \phi)$



2) The visible end of a taut string (mass m, tension T) is fixed. The opposite end (and much of the string's length) is obscured by a box which is open on both ends. The speed of sound in air is v_{snd}.

A signal generator sits nearby. As the signal generator is tuned through the range f_{lo} to f_{hi} the string is observed to vibrate at two and only two frequencies: f_a and f_b.

- 2a) (10 points) How might one use this information to determine whether the obscured end of the string is fixed or loose (without peeking)? Provide as much detail as you can.

define $\Delta f \equiv f_b - f_a$. Evaluate $f_b/\Delta f$ and $\frac{f_a}{\Delta f}$.

- If the results yield adjacent integers, Δf is the fundamental, we have "like" boundary conditions, and the obscured end is fixed.
- If the results yield two half integers, Δf is twice the fundamental, we have "mixed" boundary conditions and the obscured end is loose.

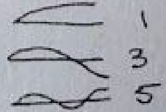
- 2b) (5 points) **For the rest of the problem, we'll suppose the obscured end is loose.** In terms of the given information, which harmonic does f_b correspond to? How many intermediate nodes (that is, nodes between the ends) are on the string when it vibrates in this mode?

(if the obscured end is loose → Mixed boundary Conditions)

The fundamental, $\mathcal{F} = \frac{\Delta f}{2} = \frac{f_b - f_a}{2}$, So $N_b = \frac{f_b}{\mathcal{F}} = \frac{2 f_b}{f_b - f_a}$

→ f_b is the $\frac{2 f_b}{f_b - f_a}$ harmonic

→ There are $\frac{1}{2}\left(\frac{2 f_b}{f_b - f_a} - 1\right)$ intermediate nodes



$2m+1 = N_b$

↑ # int nodes

$\left(\frac{1}{2} \cdot \frac{f_b + f_a}{f_b - f_a}\right)$

- 2c) (5 points) How long is the string?

Why is this 2L instead of 4L?

The fundamental, $f_s = \frac{1}{2L}\sqrt{\frac{TL}{m}} = \frac{f_b - f_a}{2}$

$$\boxed{L = \frac{T}{m(f_b - f_a)^2}}$$

??

- 2d) (5 points) Suppose that the frequencies f_a and f_b also set the box into resonance. Will there be any other frequencies within the tunable range that set the box into resonance? Explain.

The box is open on both ends → like BC → it resonates at all harmonics.
Call the fundamental frequency of the box f_B. $f_B = m(f_b - f_a)$ where m is an integer.

$$\frac{f_b}{f_a} = \frac{(2N+3)\, f_s}{(2N+1)\, f_s} = \frac{(n_a + m)\, f_B}{n_a \quad f_B}$$

{ f_a is the n_a'th harmonic in box
{ f_a is the $(2N+1)$'th harmonic in string

$$(2N+1)(n_a + m) = (2N+3)\, n_a$$

$$m = 2\left(\frac{N_A}{2N+1}\right)$$

← any integer
← ODD integer

↑ integer

To generate harmonics, $\frac{N_A}{2N+1}$ must be an integer, so m must be an even integer...

* If the box resonates, the fundamental, $f_B = \frac{f_B - f_A}{m}$ where m is an even integer. (<0) Since $f_B < f_b - f_a$ and all harmonics are present, there will be additional resonances in the box at intervals of f_B from f_a, throughout the range

- 2e) (5 points) If the box resonates at f_a and f_b, what values might its length have?

$$f_B = 2N(f_b - f_a) \qquad N = \text{INTEGER}$$

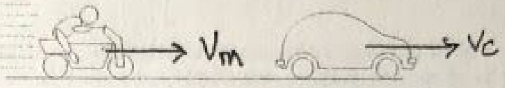
$$f_B = \frac{V_{snd}}{2L} \qquad \text{(like BC)}$$

$$\boxed{L = \frac{V_{snd}}{4N(f_b - f_a)}}$$

N is a positive integer

1
3
5
n+1 = N_b
int nodes

$$\frac{f_{obs}}{f_{src}} = \frac{V_{snd} - V_{obs}}{V_{snd} - V_{src}}$$



3) Driving along the freeway with a speed v_c, you are suddenly overtaken by a much faster motorcycle (trying desperately, no doubt, to flee pursuing raptors, but I digress). As the motorcycle approaches, you observe the dominant component of the sound emanating from its screaming occupant to have a frequency f_1. Once the motorcycle has passed (but before the raptors have caught up to you), that dominant component of the screams is observed to shift to a frequency f_2.

- 3a) (15 points) Find the speed at which the motorcycle is traveling.

$f_0 \; \bullet\!\rightarrow V_m \quad \xrightarrow{+} \quad f_1 \; \bullet\!\rightarrow V_c$

$V_{src} = +|V_m| \qquad V_{obs} = +|V_c|$

$f_2 \; \bullet\!\rightarrow V_c \quad \xleftarrow{+} \quad f_0 \; \bullet\!\rightarrow V_m$

$V_{obs} = -|V_c| \qquad V_{src} = -|V_m|$

$$\frac{f_1}{f_0} = \frac{V_{snd} - |V_c|}{V_{snd} - |V_m|} \qquad \frac{f_2}{f_0} = \frac{V_{snd} + |V_c|}{V_{snd} + |V_m|}$$

$$\frac{f_1}{f_2} = \frac{V_{snd} - |V_c|}{V_{snd} + |V_c|} \cdot \frac{V_{snd} + |V_m|}{V_{snd} - |V_m|}$$

$$f_1 (V_{snd} + |V_c|)(V_{snd} - |V_m|) = f_2 (V_{snd} - |V_c|)(V_{snd} + |V_m|)$$

$$\left[f_1 (V_{snd} + |V_c|) - f_2 (V_{snd} - |V_c|)\right] V_{snd} = |V_m| \left[f_1 (V_{snd} + |V_c|) + f_2 (V_{snd} - |V_c|)\right]$$

$$\boxed{|V_m| = V_{snd} \, \frac{(f_1 - f_2) V_{snd} + (f_1 + f_2)|V_c|}{(f_1 + f_2) V_{snd} + (f_1 - f_2)|V_c|}}$$

why is this the value we care about?

- 3b) (10 points) What frequency would that dominant component appear to have if you could match the motorcycle's speed? what is f_0?

trying
bserve
Once
of the

$$V_{snd} - |V_m| = \frac{f_0}{f_1}(V_{snd} - |V_c|)$$

$$+ \quad V_{snd} + |V_m| = \frac{f_0}{f_2}(V_{snd} + |V_c|)$$

$$2V_{snd} = f_0 \left[\frac{V_{snd} - |V_c|}{f_1} + \frac{V_{snd} + |V_c|}{f_2} \right]$$

$$\boxed{f_0 = \frac{2V_{snd} f_1 f_2}{(f_1+f_2)V_{snd} + (f_1 - f_2)|V_c|}}$$

in terms of the given quantities

V_m

$|V_m|$

Note that we only know $|V_m|$ in terms of the original speed of the car $|V_c|$, so THAT is the appropriate value for $|V_c|$. Resist the temptation to write $|V_c| = |V_m|$, because - in this case - it doesn't ..

- 3c) (5 points) Evaluate your answers to parts a and b in the limit $f_2 \to f_1$ and explain the results.

a) if $f_1 \to f_2$ $\quad |V_m| = |V_c|$

b) if $f_1 \to f_2$ $\quad f_0 = f_1 \; (= f_2)$

The only way $f_1 = f_2$ is if there is no relative motion between the cars (note - then either the first or the second picture is relevant - the motorcycle is always ahead or behind). $|V_m| = |V_c|$ and $f_1 = f_2 = f_0$ are what you'd expect in this case -