Name: SOLUTIONS

Student ID number: ____________________

Discussion section: ____________

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# MIDTERM 1
# PHYSICS 1B, WINTER 2012

## JANUARY 31, 2012

# VERSION B

## READ THE FOLLOWING CAREFULLY:

- ▷ Closed book. Calculators will not be needed, no electronic devices should be out as you take the exam. One 3″×5″ index card with notes is allowed.
- ▷ This exam consists of 9 pages (including this one) with problems numbered 1 through 3; make sure you have been given all pages/problems.
- ▷ You have 50 minutes to complete the exam.
- ▷ Make sure to write your name at the top of each page of this exam. Use the space provided on the exam pages to do your work. You may use the back of the pages also, but please mark clearly which problem you are working on (and also state underneath that problem that you have done work on the back of the page).
- ▷ Partial credit will be given. Show as much work/justification as possible (diagrams where appropriate). If you can not figure out how to complete a particular computation, a written statement of the concepts involved and qualitative comments on what you think the answer should be may be assigned partial credit.
- ▷ You must show a photo ID when turning in your exam.
- ▷ Mistakes in grading: If you find a mistake in the grading of your exam, alert the instructor within one week of the exams being returned (this will occur in discussion section following the exam date). DO NOT write on the returned graded exam – you may make a note of the problems you thought were misgraded on this first page, but any changes/additions to the subsequent pages will negate your chances for a re-grade and may result in the incident being reported to the dean of students. Note that a fraction of the graded exams will be photocopied prior to returning them to you.

[In case you need any of these constants: The speed of sound in air is 343 m/s, the speed of sound in water is 1440 m/s, the acceleration due to gravity is 9.8 m/s$^2$, $I_o$ (the reference for sound intensity) is $10^{-12}$ W/m$^2$, and the average air speed of an unladen swallow (European) is 10 m/s.]

**[1.] Short answer conceptual questions**.

**(a)** (12 pts) Two speakers are placed a distance L apart from one another in a room. They both are driven by the same audio amplifier, creating sound waves at frequency f. As you walk around the room you hear maxima and minima in sound intensity.

i. Explain briefly why the sound intensity varies at different locations in the room. What determines where a maximum will occur?

> The intensity variations are due to interference between waves coming from the two speakers. A maximum in intensity will occur when the waves from the different speakers arrive to that location in phase. This will happen if the path length difference (difference in distance traveled from one speaker compared to the other) is equal to zero or an integer multiple of the wavelength of the sound waves.

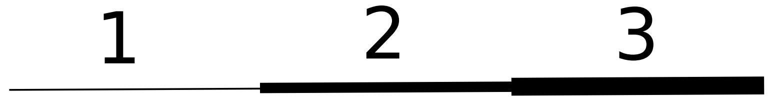
ii. You sit down at a spot which is not equidistant from each speaker, but where the intensity is maximum. The room is then filled with a different gas (you hold your breath). The speed of sound in the gas is smaller than in air. The speakers continue to broadcast at the same frequency and intensity. The intensity at your location is still a maximum after the room is filled with the new gas (you walk around to confirm). What has to be true about the speed of sound in the new gas for this to have occurred?

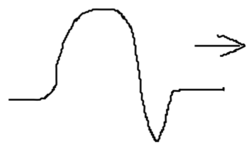
> For the location to remain a maximum, the path length difference must still be an integer multiple of the new wavelength. Let's call the old wavelength $\lambda$ and the new wavelength $\lambda'$. If there are an integer multiple of the old wavelengths in the path length difference, then $L = n\lambda$ where $L$ path length difference. If there are an integer multiple of the new wavelength in the path length difference, then it is also true that $L = n'\lambda'$, where $n'$ is not the same as $n$ (necessarily). If we equate these two (both equal to $L$) then: $n'\lambda' = n\lambda$. Noting that $\lambda f = v$ and $\lambda' f = v'$, then:
>
> $$\frac{n'}{n} = \frac{v}{v'}$$
>
> Which means that the ratio of the new speed of sound to the speed of sound in air must be a ratio of integers (e.g. 2/3 would work).

**(b)** (12 pts) You go to a sideshow at a carnival where the claim is made that a soprano can break glass with her voice. During the show, several objects made of fine crystal are put on a table including wine glasses (of various sizes), a large vase, and some candlesticks of various sizes and shapes. The soprano then sings a loud, perfect note (high "A", 880 Hz exactly), and all of the glass objects shatter. A 1B student in the audience stands up and says that there is no way that the soprano could have done that with her voice, and that the show must be a fake (i.e. the crystal objects were broken in some other manner). Do you agree or disagree with this statement, and why?

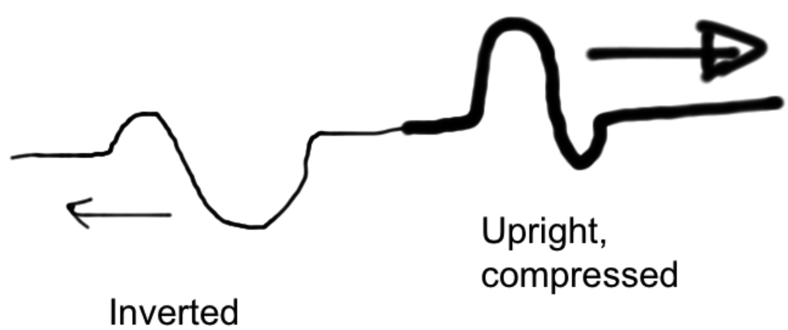
Solution: A glass object might break if the soprano hits a note which matches a resonant frequency of the object. The student is right to be skeptical – it is unlikely that all of those glass objects have a resonant frequency in common (i.e. if you thump each of them, they will likely ring with different frequencies). However, it is possible; so you can disagree with the student *if* you give the proper justification. An important aspect of this justification is that there are many resonant frequencies for these types of objects (like fixed strings or tubes; they can have many harmonics), and that this allows (in special cases) objects of differing size to have a resonant frequency in common (the fundamental of one could match a higher harmonic of another, etc).

**(c)** (12 pts) Consider the three-part rope shown below: a very long string (1) with mass density $\mu$ is tied to a short string (string 2, length $L$) with mass density $10\mu$. This second string is then tied to another very long string (3) with mass density $100\mu$.



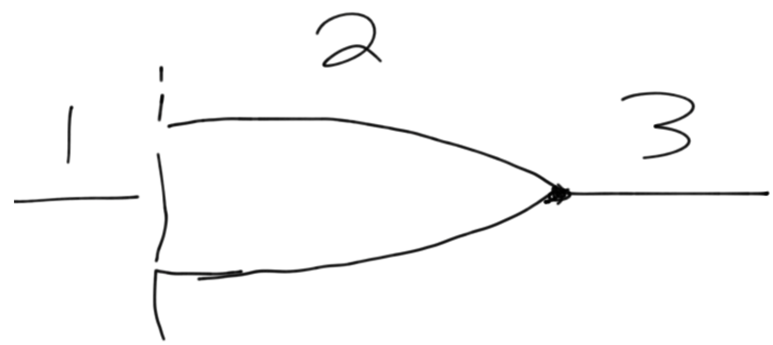
i. A wave pulse, drawn above, is traveling on string 1, approaching the knot between string 1 and 2. Draw the reflected pulse and the pulse which is transmitted onto string 2. In your drawing, pay attention to whether or not the pulses are inverted/upright but also to the overall shape of the pulse – e.g. are the reflected/transmitted pulses stretched out or compressed relative to the incident?




The reflected pulse is inverted – it acts like reflection from a fixed end since the second string is so much heavier than the first. The transmitted pulse is upright, but since the speed of waves on the heavier string is slower, the wave gets bunched up (compressed, effectively shorter wavelength).

ii. Now instead of sending a pulse down string 1, I oscillate the far end (far to the left, off the page) of string 1 sinusoidally at frequency $f$, launching sinusoidal waves down the composite string. Starting from very low frequency, I increase $f$ slowly. When I reach a certain frequency $f_1$ I notice a large amplitude standing wave gets setup on string 2. Draw this standing wave.

Because there are knots on either side of string 2, standing waves can form on the string (due to reflections at these knots causing waves to travel back and forth on string 2). The right end of string 2 acts like a fixed end for reflection (attached to a much heavier rope there), so a standing wave would have a node at that point. The left end of string 2 acts like a free end for reflection (attached to a much lighter rope there), so a standing wave would have an antinode at that point. The lowest frequency standing wave on string 2 would therefore be a 1/4 wavelength standing wave:



**[2.]** (32 pts) A block of mass $M = 2$ kg is attached to a horizontal spring of spring constant $k = 50$ N/m and is sitting at rest. A bullet of mass $m = 600$g is fired into the block and sticks. The bullet has a velocity of $v = 20$ m/s just before striking (and sticking to) the block.

**(a)** What is the frequency of the resulting oscillation?

**(b)** What is the amplitude of the resulting oscillation?

**(c)** Now an identical block is hung from a vertical spring (identical to the first). A bullet is now fired upward into the block as it hangs (at rest) from the spring (same bullet mass and velocity as before). Will your answers to (a) or (b) change in this case? If so, how (qualitative answer is enough) and why?

Solution: **(a)** The frequency of the oscillation is:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m+M}} = \frac{1}{2\pi}\sqrt{\frac{50}{2.6}} = 0.7\text{Hz}$$

**(b)** We can get the amplitude of the oscillation by finding the maximum speed of the oscillation, $v_{\text{max}} = \omega A$. The maximum speed of this oscillation is the initial velocity (the mass starts at rest and is given an initial velocity to start the motion). The inital velocity can be gotten through using momentum conservation: $mv_b = (m+M)v_o$. Where $v_b$ is the speed of the bullet and $v_o$ is the speed of the block+bullet after the (inelastic) collision.

$$v_o = \frac{m}{m+M}v_b = (20)\left(\frac{0.6}{2.6}\right) = 4.62\text{m/s}$$

The amplitude is then:

$$A = \frac{v_o}{\omega} = \frac{4.62}{(2\pi)(0.7)} = 1.0\text{m}$$

**(c)** The frequency does not change, as this just depends on the mass and the spring constant. The amplitude does change, however. The reason is that when the bullet sticks in the block, the new equilibrium point of the system is a bit lower than it was before (more mass stretches the spring a bit more in equilibrium). (Another way to think about this: if the bullet hits the block with *zero* velocity, there is still an oscillation – it is like suddenly adding mass to a hanging spring.) So,the mass starts out with a bit of potential energy (it is not at the equilibrium point) but gets the same initial kinetic energy as before. So the total energy is larger, leading to a larger amplitude.

**[3.]** (32 pts) A horizontal 1.5-m long tube, with two open ends, is mounted on a platform. The tube and platform have a combined mass of 15 kg, and undergo undamped horizontal oscillations in combination with a spring. The tube receives sound waves from a stationary loudspeaker located to the right of the oscillating system. At $t = 0$, the platform is passing through its equilibrium position, to the right, and the tube resonates in its 8th harmonic. $t = 0.2$ s is the earliest positive time at which the platform reaches its maximum displacement to the right of equilibrium, and here the tube resonates in its 6th harmonic. Use 343 m/s for the speed of sound. Define right as the positive direction.

**(a)** What is the spring constant of the spring to which the platform is attached?

**(b)** What is the frequency of the sound waves emitted by the loudspeaker?

**(c)** What is the speed with which the platform passes through equilibrium?

**(d)** What are the displacements of the platform, while moving to the right, when the tube resonates in its 7th harmonic? What is the earliest time at which this occurs?

**(e)** Write an expression for the time-dependence of the frequency of the sound waves received by the tube. What is the lowest harmonic experienced by the tube during the oscillations? Justify your answer.

**Solution: part a:** The period is 4 times the time from equilibrium to maximum displacement, or $T = (4)(0.2) = 0.8$ s. The spring constant is $k$, where

$$2\pi\sqrt{\frac{m}{k}} = T \tag{1}$$

$$\omega = \frac{2\pi}{T} = \frac{5\pi}{2}\ \mathrm{s}^{-1} \tag{2}$$

$$k = m\omega^2 = \frac{(15)(25)\pi^2}{4}\ \mathrm{N/m} = 925\ \mathrm{N/m} \tag{3}$$

**part b:** When the platform is at maximum displacement, the tube is not moving, and therefore receives sound waves at the same frequency as the frequency emitted by the speaker. Because the tube is open and resonates in its 6th harmonic, this frequency is

$$f_6 = \frac{nv}{2L} = \frac{(6)(343)}{(2)(1.5)} = 686\ \mathrm{Hz} \tag{4}$$

**part c:** Let $v_m$ denote the velocity through equilibrium. The frequency $f_8$, detected when the platform moves through equilibrium, to the right, is doppler-shifted from the speaker frequency $f_6$ according to

$$f_8 = (f_6)\frac{v + v_m}{v} \tag{5}$$

$$v_m = v\left(\frac{f_8}{f_6} - 1\right) = (343)\left(\frac{8}{6} - 1\right) = \frac{343}{3} = 114.3 \text{ m/s} \tag{6}$$

The platform moves through equilibrium at 114.3 m/s.

**part d:** The platform velocity at the 7th harmonic is $v_7$, where

$$f_7 = (f_6)\frac{v + v_7}{v} \tag{7}$$

$$v_7 = v\left(\frac{f_7}{f_6} - 1\right) = (343)\left(\frac{7}{6} - 1\right) = \frac{343}{6} = 57.2 \text{ m/s} = \frac{v_m}{2} \tag{8}$$

The displacements $x_7$, corresponding to the 7th harmonic, are given by

$$\frac{1}{2}kx_7^2 + \frac{1}{2}mv_7^2 = \frac{1}{2}mv_m^2 \tag{9}$$

$$x_7 = \frac{\pm\sqrt{v_m^2 - v_7^2}}{\omega} = \pm\frac{2}{5\pi}\frac{\sqrt{3}}{2}v_m = \pm 12.6 \text{ m} \tag{10}$$

The platform passes through $x = 0$ at $t = 0$, and is moving to the right, in the positive direction. Therefore, the time-dependence of the velocity is

$$v(t) = v_m \cos(\omega t) \tag{11}$$

The 7th harmonic is first reached at a time $t_7$, given by

$$v_7 = v_m \cos(\omega t_7) \tag{12}$$

$$\omega t_7 = \cos^{-1}\left(\frac{v_7}{v_m}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \tag{13}$$

$$t_7 = \frac{\pi}{3\omega} = \frac{2}{15} = 0.133 \text{ s} \tag{14}$$

**part e:** The frequency arriving at the tube has the following time-dependence:

$$f(t) \;=\; (f_6)\frac{v + v_{\text{tube}}(t)}{v} \;=\; (f_6)\frac{v + v_m\cos(\omega t)}{v} \tag{15}$$

where $v = 343$ m/s is the sound speed. The lowest harmonic is reached when the platform is moving to the left through equilibrium (in the negative $x$-direction). The resulting downward shift in frequency is the same as the upward shift in frequency that occurs when the platform moves through equilibrium in the positive direction. The upward shift is from the 6th harmonic to the 8th harmonic, so the downward shift is from the 6th harmonic to the 4th harmonic. Or, mathematically, we have

$$f_8 \;=\; (f_6)\left(1 + \frac{v_m}{v}\right) \tag{16}$$

$$\frac{v_m}{v} \;=\; \frac{f_8}{f_6} - 1 \tag{17}$$

$$f_{\text{min}} \;=\; (f_6)\left(1 - \frac{v_m}{v}\right) \;=\; (f_6)\left(2 - \frac{f_8}{f_6}\right) \;=\; \frac{2f_6}{3} \;=\; \left(\frac{2}{3}\right)\frac{6v}{2L} \;=\; \frac{4v}{2L} \tag{18}$$

So, the 4th harmonic is the lowest one experienced by the tube during the oscillation.