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Physics 1AH – Prof. J. Rosenzweig – Fall 2017

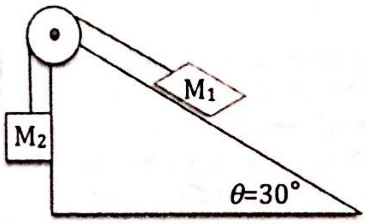
Midterm 1

October 24, 2017

Use only the paper provided for you. Show all of your work for full credit. Write your name on each sheet of paper in your answers, then staple all together in order. You have 1 hour and 50 minutes to complete this exam.

You are permitted one sheet of paper as notes, with writing on both sides.

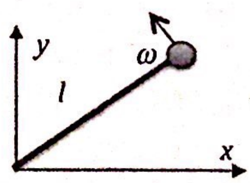
1. Consider the system of a two masses connected by a cable of negligible mass which passes over a pulley. One of the masses is freely hanging, forced downward by gravity, the other is on the declining slope of a fixed wedge, which has an angle of 30°, as shown below. The coefficient of static friction $\mu=0.25$.



   (a) Diagram the forces, including tension and normal forces. (10 pts)
   (b) What are the constraints to the motion? (5 pts)
   (c) Assume $M_1=2$ kg, what is the maximum value of M_2 permitted before the masses move? (10 pts)

2. An object of mass M rotates in the (x,y) plane with angular frequency ω about a fixed point, constrained by a string of negligible mass and length l.

   only give position & velocity
   (a) Describe the motion of the mass in Cartesian coordinates. (5 pts)
   (b) Describe the motion of the mass in polar coordinates. (5 pts)
   (c) What is the tension in the string? (10 pts)
   (d) If the string breaks when the mass passes $\theta=0$, describe the subsequent motion of the mass. (5 pts)



3. Consider a spring of negligible mass having spring constant k that hangs from a ceiling. Its bottom end is located at y_0. An object of mass M is hung from the spring as shown below

(a) Where would the new equilibrium position of the end of the spring y_{eq}, where the spring does not move? (10 pts)

(b) As the mass is suddenly hung from the spring, the spring end, while initially not moving, is not at this equilibrium position, but is at y_0. Write an expression for the oscillatory motion of the spring end, given these initial conditions. (15 pts)

rest

4. Her Majesty's agent, James Bond, is scuba diving while attempting to place a bomb on the hull of a large boat belonging to evil mastermind Goldfinger's fleet. Goldfinger's henchmen on the boat are shooting bullets directly downward (in direction y) into the water in an attempt to neutralize Bond. The water is a highly viscous medium, however, and Bond realizes that if he dives, the bullets will not harm him. The bullet's mass m=0.05 kg, its velocity at gun exit is 500 m/sec; a safe value of the velocity as the bullets may strike Bond is 50 m/sec.

Write diff EQ.

(a) The viscous force may be written as $\vec{F} = -C\vec{v}$, where for the bullet in water the constant C=5 kg-sec^{-1}. Ignoring the effects of gravity, write the equation of motion for the velocity of the bullet. (10 pts)

Solution to Diff EQ in (a)

(b) Recognizing that the solution to this equation is proportional to an exponential having the form $\exp(-t/\tau)$, where τ is a characteristic time given by m and C, write velocity as a function of time t. (5 pts)

(c) What is the solution for the depth $y(t)$? (10 pts)

(d) **Extra credit.** What is the minimum depth must Bond swim at in order to stay safe? (10 pts)

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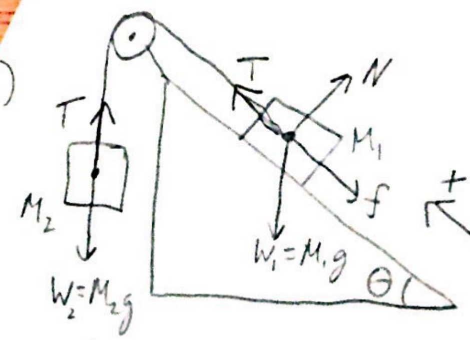
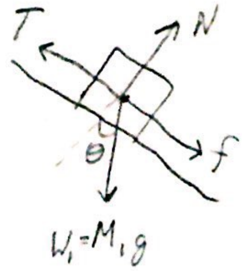
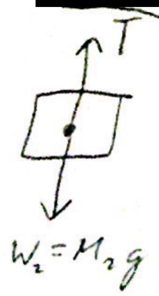
a)



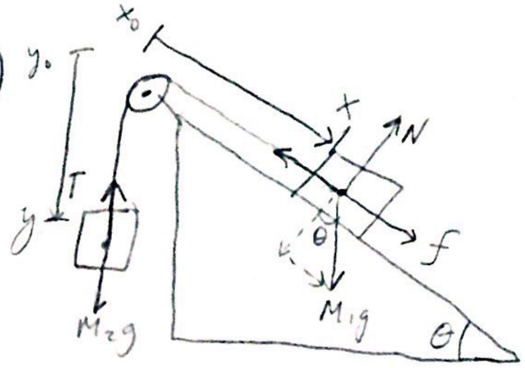
M_1 :



M_2 :



b)



Constraint EQ:

$$(x_0 - x) + (y - y_0) = \ell \text{, where } \ell = \text{const}$$

$$-\ddot{x} + \ddot{y} = 0$$

$$\boxed{\ddot{x} = \ddot{y}}.$$

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c) For M_1: $N - M_1 g\cos\theta = 0$ } For M_1, $\Sigma F_y = 0$

$$N = M_1 g\cos\theta$$

$$M_1\ddot{x} = T - f - M_1 g\sin\theta \text{, where } f = \mu N$$

$$M_1\ddot{x} = T - \mu M_1 g\cos\theta - M_1 g\sin\theta \quad (1)$$

For M_2: $M_2\ddot{y} = M_2 g - T \quad (2)$

When they don't move, $\ddot{x} = \ddot{y} = 0$. Thus,

$$T = \mu M_1 g\cos\theta + M_1 g\sin\theta \quad (3)$$

$$T = M_2 g \quad (4)$$

(3) = (4): $M_2 \cancel{g} = \mu M_1 \cancel{g}\cos\theta + M_1 \cancel{g}\sin\theta$

$$\boxed{M_2 = \mu M_1\cos\theta + M_1\sin\theta}$$

Plugging in known values,

$$M_2 = \frac{1}{4}(2\text{ kg})\cos 30^\circ + (2\text{ kg})\sin 30^\circ$$

$$= \left[\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{2}\right)\right]\text{kg}$$

$$\boxed{M_2 = \frac{\sqrt{3}+4}{4}\text{ kg} \approx 1.433\text{ kg}}$$

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2) a)

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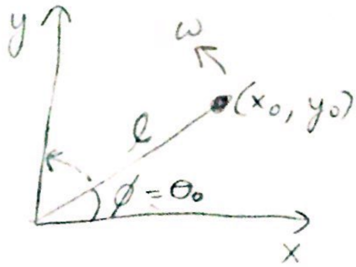


Cartesian:

$$\left.\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right\} \text{ where } r = \ell$$

$$\vec{r} = \vec{x}\hat{\imath} + \vec{y}\hat{\jmath}$$

$$\theta_0 = \phi = \tan^{-1}\left(\frac{y_0}{x_0}\right), \ \theta = \omega t + \theta_0 = \omega t + \phi$$

$$\vec{r} = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath}$$

$$\boxed{\vec{r} = \ell\cos(\omega t + \phi)\hat{\imath} + \ell\sin(\omega t + \phi)\hat{\jmath}}$$

$$\frac{d\vec{r}}{dt} = \boxed{\vec{v} = -\omega\ell\sin(\omega t + \phi)\hat{\imath} + \omega\ell\cos(\omega t + \phi)\hat{\jmath}}$$

b) In polar, $\vec{r} = r\hat{r}$, where $r = \ell$ (constant)

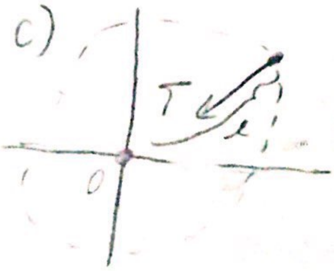
$$\boxed{\vec{r} = \ell\hat{r}}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$= 0\hat{r} + \ell\omega\hat{\theta}$$

$$\boxed{\vec{v} = \ell\omega\hat{\theta}}$$

c)



$$-T = \sum F_r$$

$$-T = ma_r$$

$$-T = m(\ddot{r} - r\dot{\theta}^2)\hat{r}, \text{ where } r = \ell \text{ (const)}, \dot{r} = \ddot{r} = 0, \ \&\ \dot{\theta} = \omega$$

$$-T = m\ell\dot{\theta}^2$$

O.K. $\boxed{T = +m\ell\omega^2, \text{ pointing radially inward}}$

d) If the string breaks when $\theta = 0$, the mass will move tangentially from where it broke. Thus, it will move parallel to the y-axis with a velocity of $\vec{v} = \omega\ell\hat{\jmath}$.

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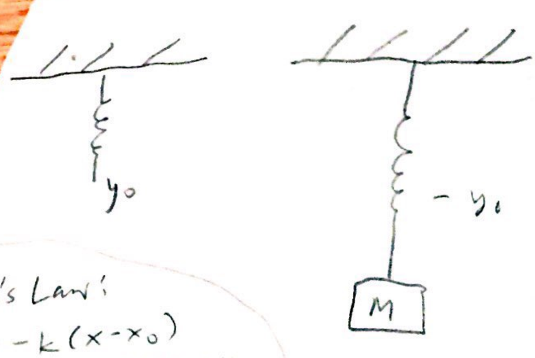

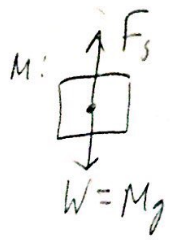

$$\sum F = F_s - W = 0$$

$$0 = -k(y_0 - y_{eq}) - Mg$$

$$Mg = -k(y_0 - y_{eq})$$

$$-\frac{M}{k}g = y_0 - y_{eq}$$

$$\boxed{y_{eq} = y_0 + \frac{M}{k}g}$$

* Hooke's Law:
$F_s = -k(x - x_0)$
$\hookrightarrow x_0 = x_{eq}$

b) Initial conditions: $y(0) = y_0$, $\dot{y}(0) = 0$

Using init. conditions, we can find A & B for:

$$y = A\cos(\omega_0 t) + B\sin(\omega_0 t) + y_{eq}$$

$$\Rightarrow \quad y(0) = A + y_{eq} = y_0$$

$$\therefore A = y_0 - y_{eq} = y_0 - \left(y_0 + \frac{M}{k}g\right) = -\frac{M}{k}g = A$$

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$$\dot{y} = -A\omega_0\sin(\omega_0 t) + B\omega_0\cos(\omega_0 t)$$

$$\dot{y}(0) = B\omega_0 = 0 \Rightarrow B = 0$$

$$\Rightarrow \quad y = -\frac{M}{k}g\cos(\omega_0 t) + y_0 - \frac{M}{k}g$$

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For springs, we know the frequency is $\omega = \sqrt{\frac{k}{M}}$. Thus,

$$\boxed{y = y_0 + \frac{M}{k}g - \frac{M}{k}g\cos\left(\sqrt{\frac{k}{M}}\,t\right)}$$

4) a) $\vec{F} = -C\vec{v} = m\vec{a} = m\frac{d\vec{v}}{dt}$

$$\Rightarrow \boxed{-\frac{C}{m}\vec{v} = \frac{d\vec{v}}{dt}}$$

$$\frac{\cancel{t}}{\tau} = \frac{-C\cancel{t}}{m}$$

$$\tau = \frac{m}{C}$$

b)

$$\frac{d\vec{v}}{dt} = -\frac{C}{m}\vec{v}$$

$$\int_{v_0}^{v}\frac{1}{v}\,dv = -\frac{C}{m}\int_0^t dt$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{C}{m}t$$

$$\frac{v}{v_0} = \exp\left[-\frac{C}{m}t\right], \text{ where } \tau = \frac{m}{C}$$

$$\boxed{v = v_0\exp\left[-\frac{C}{m}t\right] = v_0\exp\left[-\frac{t}{\tau}\right]}$$

[redacted]

4) c) $y(t) = \int v\,d\tilde{t} = \int_0^t v_0 \exp(\frac{-\tilde{t}}{\tau})\,d\tilde{t}$

$* y_0 = 0$ so
$y(t) = y_0 + \int v\,dt$
$= \int v\,dt *$

$= v_0 \int_0^t e^{(-\frac{1}{\tau})\tilde{t}}\,d\tilde{t}$

$= v_0(-\tau)e^{(-\frac{1}{\tau})\tilde{t}}\Big|_0^t$

$= -v_0\tau\left[e^{-\frac{t}{\tau}} - 1\right]$

$= v_0\tau\left[1 - e^{-\frac{t}{\tau}}\right]$

$$\boxed{y(t) = v_0\tau\left[1 - \exp(\frac{-t}{\tau})\right],\ \text{where } \tau = \frac{m}{c}}$$

d) $v = v_0 \exp\left[\frac{-c}{m} t\right]$

$\hookrightarrow \ln\left(\frac{v}{v_0}\right) = -\frac{c}{m} t$

$t = -\frac{m}{c} \ln\left(\frac{v}{v_0}\right)$

$= -\frac{.05\,kg}{5\,kg \cdot sec^{-1}} \ln\left(\frac{50\,m/s}{500\,m/s}\right)$

$t = .023\,s$

$\Rightarrow y(.023\,s) = (500\,m/s)\left(\frac{.05\,kg}{5\,kg \cdot sec^{-1}}\right)\left(1 - \exp\left[\frac{-.023\,s}{\left(\frac{.05\,kg}{5\,kg \cdot sec^{-1}}\right)}\right]\right)$

$\approx \boxed{4.50\,m}$

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