MIDTERM 2 – PHYSICS 1A – WINTER 2012

Name:______________________________

Student ID No.:______________________________

Signature:______________________________

Specify here if you want your exam returned privately:________

Each question is worth 25 points. The exam is closed book: no notes or calculators. If necessary, use the back of the page. If you do use the back of the page, write OVER on the page whose backside you are using. Please make the organization of your answer as clear as possible.

1. Assume that there is a repulsive force between m_1 which is at rest and an incident particle of mass m_2, incoming on a radial trajectory. Assume that $m_1 >> m_2$. Assume the potential energy is of the form:

$$U(r) = K e^{r_0/r} \tag{1}$$

(1) What is the force associated with this potential energy? (10 pts.) (2) Very far from the central mass, the incoming particle has speed v_0, how close can it approach the central mass? (15 pts.)

$\vec{F} = -\frac{du}{dr}\hat{r}$ $\Rightarrow$ $\boxed{\vec{F} = \frac{K r_0}{r^2} e^{r_0/r}\hat{r}}$

Note at $r = \infty$ $u = K$

$\Rightarrow K + \frac{1}{2}mv^2 = Ke^{r_0/r}$ for closest approach

$\Rightarrow \left(1 + \frac{mv^2}{2K}\right) = e^{r_0/r}$ $\Rightarrow$ $\frac{r_0}{r} = \ln\left(1 + \frac{mv^2}{2K}\right)$

$\Rightarrow \boxed{r = \frac{r_0}{\ln\left(1 + \frac{mv^2}{2K}\right)}}$

2. Assume a new law of gravity where

$$\vec{F} = -\frac{G' M_1 M_2}{R^5}\hat{r} \tag{2}$$

(1) What are the units of G'? (5 pts.) (3) For a test mass in a circular orbit, what is the relationship between its kinetic energy and its potential energy? (10 pts.) (4) What would be the form of Kepler's third law with this new law of gravity? (10 pts.)

2. $kg\, m\, s^{-2} = G' \frac{kg^2}{m^5} \Rightarrow \boxed{G' = \frac{[m^6]}{[kg\, s^2]}}$

$\frac{v^2}{R} = \frac{GM}{R^5} \Rightarrow \frac{1}{2} v^2 = \frac{GM}{R^4}$ $\quad u = -\frac{GM}{4R^4} m$

$\Rightarrow \boxed{\frac{1}{2} m v^2 = T = 2|u|}$

$\frac{v^2}{R} = \frac{GM}{R^5}$ $\quad v = \frac{2\pi R}{P} \Rightarrow \frac{4\pi^2 R^2}{R P^2} = \frac{GM}{R^5}$

$\Rightarrow \boxed{P^2 = \frac{R^6\, 4\pi^2}{GM}}$

3. Consider a radioactive decay where a particle at rest decays into two particles of mass m_1 and m_2. Let E denote the energy released during this radioactive decay and let $\vec{u}_1$ and $\vec{u}_2$ denote the velocities of particles 1 and 2 after the decay. What is the expression for conservation of momentum after the decay? (5 pts.) What is the expression for conservation of energy after the decay? (5 pts.) Compute the kinetic energy of each particle after the decay in terms of E, m_1 and m_2 only (15 pts.).

$$m_1 \vec{u}_1 + m_2 \vec{u}_2 = 0$$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = E$$

Use $m_1 u_1 = -m_2 u_2 \Rightarrow u_1^2 = \left(\frac{m_2}{m_1}\right)^2 u_2^2$

$$\Rightarrow \frac{1}{2} m_1 \left(\frac{m_2}{m_1}\right)^2 u_2^2 + \frac{1}{2} m_2 u_2^2 = E \Rightarrow u_2^2 = \frac{2E\, m_1}{m_2(m_2+m_1)}$$

$$\Rightarrow \boxed{T(\#2) = \frac{1}{2} m_2 u_2^2 = \frac{E\, m_1}{m_1+m_2}}$$

$\Rightarrow$ By conservation of energy

$$\boxed{T(\#1) = \frac{E\, m_2}{m_1+m_2}}$$

4. Assume that Earth has radius R_E and is homogeneous with density ρ. (1) If $R > R_E$, what is the force of gravity on a test particle with mass, m_{test}? (5 pts.). (2) If $R < R_E$, what is the force of gravity on this test mass? (5 pts.) (5 pts.) (3) Assuming there is a tunnel through Earth, what would be the escape velocity from the center where $R = 0$? Express your result in terms of G, the mass of Earth, M_E and R_E. (15 pts.)

$$\vec{F} = -\frac{\frac{4\pi}{3} G \rho R_E^3 m_{test}}{R^2} \hat{r} \qquad R > R_E$$

$$\vec{F} = -\frac{4\pi}{3} G \rho R m_{test} \hat{r} \qquad R < R_E$$

interior work $\int_0^{R_E} \frac{4\pi}{3} G \rho R m_{test} \, dR = \frac{2\pi}{3} G \rho R_E^2 m_{test}$

$$= \frac{M_E G}{2 R_E} m_{test}$$

exterior work: $\int_{R_E}^{\infty} \frac{4\pi}{3} G \rho \frac{R_E^3 m_{test}}{R^2} dR = \frac{4\pi}{3} \rho G R_E^2 m_{test}$

$$= \frac{M_E G}{R_E} m_{test}$$

$$\frac{1}{2} m_{test} v_{esc}^2 = \left(\frac{M_E G}{2 R_E} + \frac{M_E G}{R_E} \right) m_{test}$$

$$\Rightarrow v_{esc}^2 = \frac{3 M_E G}{R_E} \quad \text{or} \quad v_{esc} = \left(\frac{3 M_E G}{R_E} \right)^{1/2}$$