MIDTERM 1 – PHYSICS 1A – FALL 2005

Name:________________________________

Student ID No.:________________________________

Signature:________________________________

Specify here if you want your exam returned privately:________

Each question is worth 25 points. The exam is closed book; no notes or calculators. If necessary, use the back of the page. If you do use the back of the page, write OVER on the page whose backside you are using. Please make the organization of your answer as clear as possible.

1. Consider an object of mass m that starts at rest at the top of a frictionless ramp of height h that makes angle θ with respect to the horizontal. Let Z denote the vertical axis, assume that $z = 0$ at the top of the ramp and that z increases downwards. (i) If there is no air resistance, what is the acceleration of the mass along the ramp (5 pts)? (ii) Compute the speed as a function of time, $v(t)$, as the object moves down the ramp (5 pts.) (iii) Compute the distance, D, the mass travels along the ramp as a function of time (5 pts.) (iv) Compute v as a function of z as the object moves down the ramp (10 pts.)

5 (i) $a = g\sin\theta$

5 (ii) $v = [g\sin\theta]\,t$

5 (iii) $D = \frac{1}{2}[g\sin\theta]\,t^2$

10 (iv) $z = D\sin\theta \Rightarrow t = \left(\frac{2z}{g}\right)^{1/2}\frac{1}{\sin\theta}$

$\Rightarrow v = (2gz)^{1/2}$

2. Consider the motion of a test particle moving in the X, Y plane. It starts at rest at the origin (0,0) and accelerates as:

$$\vec{a} = K_1 t^2 \hat{x} + K_2 t \hat{y} \tag{1}$$

where t is the time and K_1 and K_2 are constants. (i) What are the units of K_1 and K_2? (5 pts.) (ii) What is the velocity as a function of time? (5 pts.)? (iii) What is the location as a function of time? (5 pts.) (iv) Write down the expression for the distance of the object from the origin as a function of time. (5 pts.) (v) For $t > 0$, determine if there is ever a time when the acceleration is perpendicular to the velocity. BRIEFLY explain your answer.(5 pts).

5 (i) $K_1 : [m\,s^{-4}]$ $K_2 : [m\,s^{-3}]$

5 (ii) $\vec{v} = \frac{K_1 t^3}{3}\hat{x} + \frac{K_2}{2} t^2 \hat{y}$

5 (iii) $\vec{r} = \frac{K_1 t^4}{12}\hat{x} + \frac{K_2}{6} t^3 \hat{y}$

5 (iv) $|\vec{r}| = \left[\frac{K_1^2 t^8}{144} + \frac{K_2^2 t^6}{36}\right]^{1/2}$

5 (v) $\vec{a} \cdot \vec{v} = \frac{K_1^2 t^5}{3} + \frac{K_2^2 t^3}{2}$ always > 0

$\Rightarrow$ never perpendicul

3. (i) Write down the Equation of Hydrostatic Equilibrium (5 pts.) (ii) Show that the dimensions on both sides of the Equation you have written down are the same (5 pts.) Now assume that this Equation applied to the solid Earth and that:

$$p = A\rho_0 + B\rho^2 \tag{2}$$

where ρ_0 is the density at the surface of the Earth taken as $z = 0$. What are the units of A and B? (5 pts.) (iv) Solve for ρ as a function of z. (10 pts.)

5 (i) $\frac{dp}{dz} = -\rho g$

5 (ii) L.H.S. : $[kg\, m\, s^{-2}] \cdot m^{-2} \cdot m^{-1} = kg\, m^{-2} s^{-2}$

R.H.S. : $[kg\, m^{-3}][m\, s^{-2}] = kg\, m^{-2} s^{-2}$

5 (iii) A : $[m^2 s^{-2}]$ B: $[kg^{-1} m^5 s^{-2}]$

10 (iv) $\frac{dp}{dz} = 2B\rho \frac{d\rho}{dz} \Rightarrow 2B \frac{d\rho}{dz} = -g$

$\Rightarrow \rho = \rho_0 - \frac{g}{2B} z$

4. A ball of mass m is dropped from height h and it is subject both to the force of gravity, and air resistance which acts as $-b\vec{v}$ where $\vec{v}$ is the velocity vector of the ball. (i) What are the units of b? (5 pts.) (ii) Using Newton's 2nd Law, write down an equation that governs the speed of the ball. (5 pts.) (iii) If it starts from rest, find the ball's speed as a function of time. (10 pts.)

Hint: One possible approach to this part of the problem would be to define a new variable, y, such that:

$$y = \frac{b}{m}v - g \tag{3}$$

Solve for y as a function of t and then find v as a function of t making sure that $v = 0$ at $t = 0$.

(iv) Show that during "short" times just after the ball is released, air resistance is not important (5 pts).

5 (i) $[ma] = -[bv] \Rightarrow b = [kg\ s^{-1}]$

5 (ii) $m\frac{dv}{dt} = mg - bv$ or $\frac{dv}{dt} = g - \frac{b}{m}v$

10 (iii) with substitution $\frac{m}{b}\frac{dy}{dt} = -y$

$\Rightarrow y = y_0 e^{-tb/m} \Rightarrow \frac{b}{m}v = g + y_0 e^{-tb/m}$

choose $y_0 = -g \Rightarrow v = \frac{m}{b}g(1 - e^{-tb/m})$

5 (iv) If $t \ll m/b \Rightarrow e^{-tb/m} \simeq 1 - \frac{tb}{m}$

$\Rightarrow v \simeq gt$ independent of b