

MT1 Physics 1A(3), W17

Full Name (Printed) ~~XXXXXXXXXX~~

Full Name (Signature) ~~XXXXXXXXXX~~

Student ID Number ~~XXXXXX~~

Seat Number _____

Problem	Grade
1	25/30
2	19/30
3	20/30
Total	70/90

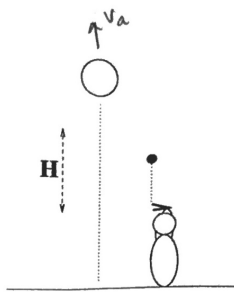
Average: 47/90

std dev: ~17

Your percentile: ~85

(top 25 / ~175 students)

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



1) A small hot-air balloon slowly rises from the surface of the Earth at a constant speed v_a . Nearby, a young child holds a loaded slingshot above his head, pointed straight up. When the balloon reaches a height H above the slingshot, the child fires a marble with a large velocity v_b along a vertical path adjacent to that of the balloon.

+5/10 • 1a) (10 points) How fast is the marble moving (relative to the child) when it first overtakes the balloon?

$$v_a > v_b \quad \text{relative velocity of marble} = v_b - v_a$$

$$a_a = 0, \quad a_b = -g \quad y - y_0 = v_0 t + \frac{1}{2} a t^2$$

$$H = (v_b - v_a)t - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 + (v_a - v_b)t + H = 0$$

$$t = \frac{(v_b - v_a) \pm \sqrt{(v_a - v_b)^2 - 2gH}}{g}$$

first one = negative sign \Rightarrow

$$t_1 = \frac{(v_b - v_a) + \sqrt{(v_a - v_b)^2 - 2gH}}{g}$$

+10/10

• 1b) (10 points) How far above the balloon will the marble appear to go?

Find: d when $v_b - v_a = 0$

$$v_f^2 = v_0^2 + 2ad$$

$$(v_b - v_a)^2 = (v_b - v_a)^2 - 2gd$$

$$2gd = (v_b - v_a)^2$$

$$d = \frac{(v_b - v_a)^2}{2g}$$

d is distance marble travelled, so subtract H

$$d = \frac{(v_b - v_a)^2}{2g} - H$$

- +5/5 • 1c) (5 points) How fast is the marble moving (relative to the child) when the marble reaches its greatest distance above the balloon. Explain the relevance of your answer.

The marble is moving at the same speed as the balloon (v_b). At this point, the marble's relative velocity to the balloon is 0.

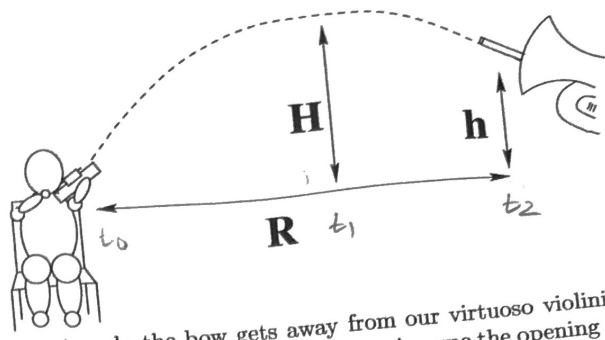
After this point, since the acceleration a_a is negative, the marble's relative velocity to the balloon will become negative and the distance between them will decrease.

- +5/5 • 1d) (5 points) How much time will elapse between the marble's first encounter with the balloon and its last?

From part a: $t = \frac{(v_b - v_a) \pm \sqrt{(v_a - v_b)^2 - 2gH}}{g}$

$$\Delta t = \frac{(v_b - v_a) + \sqrt{(v_a - v_b)^2 - 2gH}}{g} - \frac{(v_b - v_a) + \sqrt{(v_a - v_b)^2 - 2gH}}{g}$$

$$= \boxed{\frac{2\sqrt{(v_a - v_b)^2 - 2gH}}{g}}$$



2) During a particularly lively solo the bow gets away from our virtuoso violinist. It rises to a maximum height H above the violin then descends into a nearby tuba. Assume the opening of the tuba lies a horizontal distance R from and a vertical height h above the violin and answer the following questions...

• 2a) (10 points) For how long was the bow in flight?

$R = v_0 \cos \theta t_2$
 $v_f^2 = v_0^2 + 2ad + 2$
 $\Delta y = v_0 y t + \frac{1}{2} a t^2$
 $v_f = v_0 + at$
 $t_2 = \frac{R}{v_0 \cos \theta}$

$h - H = -\frac{1}{2} g (t_2 - t_1)^2 + 2$
 $H - h = \frac{1}{2} g (t_2 - t_1)^2$

$t_2 - t_1 = \sqrt{\frac{2(H-h)}{g}}$

$t_0 = 0$	t_1	t_2
$y_0 = 0$	$y_1 = H$	$y_2 = h$
$v_0 = 0$	$x_1 = R$	$x_2 = R$
$v_0 = ?$	$v_0 = 0$	$v_2 = ?$

$h = v_0 \sin \theta t_2 - \frac{1}{2} g t_2^2 + 1$

$R \tan \theta = h + \frac{1}{2} g t_2^2$

$t_2 = \sqrt{\frac{2(R \tan \theta - h)}{g}}$

Let $t_2 =$ time of flight

5

• 2b) (10 points) With what speed did the bow leave the violin?

$v_f = v_0 + at$
 $0 = v_0 \sin \theta - g(t_1)$
 $v_0 \sin \theta = g(t_1)$

$v_f^2 = v_0^2 + 2ad$
 $R = v_0 \cos \theta t_2$

$v_0 \cos \theta = \frac{t_2}{R} + 2$

$v_0 \sin \theta = \frac{h + \frac{1}{2} g t_2^2}{t_2}$ (from part a) + 2

$v_0 = \sqrt{(v_0 \sin \theta)^2 + (v_0 \cos \theta)^2}$
 $= \sqrt{\left(\frac{t_2}{R}\right)^2 + \left(\frac{h + \frac{1}{2} g t_2^2}{t_2}\right)^2}$

where $t_2 =$ time of flight from part a

10

- 2c) (10 points) At what angle with respect to the horizontal did the bow enter the tuba?

using
 $t_2 = \text{time of flight}$
 from part a

$$t_2 = \sqrt{\frac{2(R \cos \theta - h)}{g}}$$

$$\theta = \arctan\left(\frac{h + \frac{1}{2}gt_2^2}{R}\right)$$

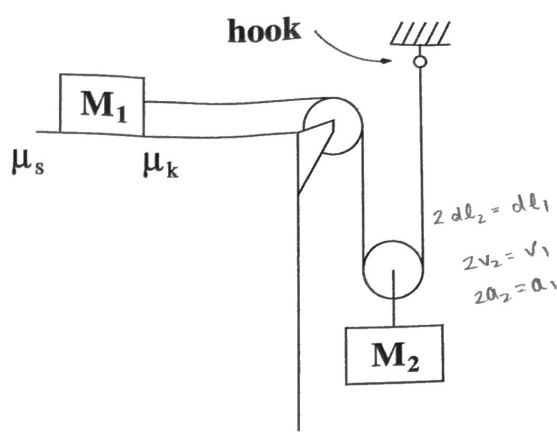
$$v_2 \cos \phi = v_0 \cos \theta$$

$$(v_2 \sin \phi)^2 = 2g(H-h) + 2$$

$$v_2 = \sqrt{(v_2 \cos \phi)^2 + 2g(H-h)^2}$$

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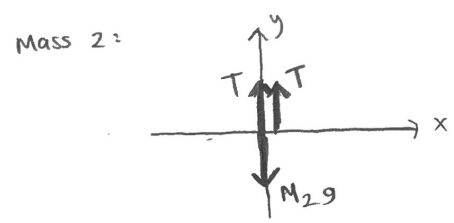
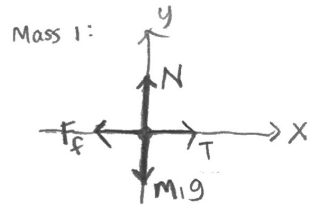
(4)



Consider the apparatus shown above. The coefficients of friction between block one and the table are both known, as are the masses M_1 and M_2 .

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- 3a) (5 points) Identify the object (or objects) you're interested in, and draw free-body diagrams for each.



- 3b) (10 points) Use Newton's laws to obtain equations that describe the dynamics of each of the objects of interest. Describe friction as F_f , and allow for acceleration of the body or bodies.

$$T - F_f = m_1 a_{1x}$$

$$N - m_1 g = m_1 a_{1y}$$

$$2T - M_2 g = M_2 a_{2y}$$

$$M_2 a_x = 0$$

Normal force \neq

- 3c) (10 points) What is the acceleration of each block?

$$T - F_f = m_1 a_{1x} \quad 2T - m_2 g = m_2 a_{2y}$$

$$N - m_1 g = m_1 a_y = 0 \Rightarrow N = m_1 g$$

$$F_f = \mu_k N = \mu_k m_1 g$$

$$T - \mu_k m_1 g = m_1 a_{1x}$$

$$2T - m_2 g = m_2 a_{2y}$$

$$2 \, dl_2 = dl_1$$

$$2v_2 = v_1$$

$$2a_2 = a_1$$

$$2a_{2y} = a_{1x}$$

$$T - \mu_k m_1 g = (m_1) 2a_{2y} \Rightarrow T = \mu_k m_1 g + 2m_1 a_{2y}$$

$$2T - m_2 g = m_2 a_{2y} \Rightarrow 2(\mu_k m_1 g + 2m_1 a_{2y}) = m_2 a_{2y}$$

$$2\mu_k m_1 g = a_{2y} (m_2 - 4m_1)$$

$$a_{2y} = \frac{2\mu_k m_1 g}{m_2 - 4m_1}$$

$$a_{1x} = \frac{\mu_k m_1 g}{m_2 - 4m_1}$$

10

- 3d) (5 points) How much force must the hook exert on the rope?

↪ The rope pulls the hook down with a force T , so the hook must pull with an equal and opposite force.

$$T = \mu_k m_1 g + 2m_1 a_{2y}$$

$$= \mu_k m_1 g + 2m_1 \left[\frac{2\mu_k m_1 g}{m_2 - 4m_1} \right]$$