$\frac{+80}{100}$

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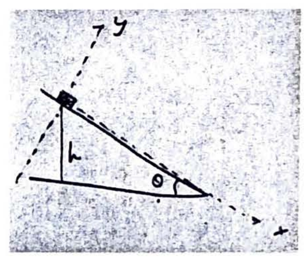
Physics 1AH

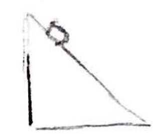
Instructions: you may bring a sheet of notes (using both sides of the paper) to the exam. Calculators are okay, but please use only basic arithmetic functions. Please show your work on each problem.

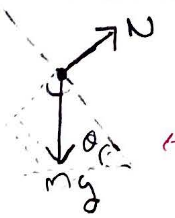
Problem 1 29 *(30 points)*

An object of mass m slides down a frictionless plane, which is inclined at an angle θ with respect to the horizontal. Assume that it starts from rest at height h from the ground.

(a) Placing your coordinate system so that the x-axis is along the plane, and the origin is at the starting position of the object, find the equations of motion. Calculate the kinetic and potential energies of the system, as functions of time. Show that at any time, their sum is equal to a constant. *(10 points)* +10







90 - θ
← +3 →
90 - 90

$F_x = mg\sin\theta = ma_x \qquad a_x = g\sin\theta$

$F_y = 0 \qquad a_y = 0$

so $[a_x = g\sin\theta] \qquad V_x = V_0 + g\sin\theta t = \underbrace{g\sin\theta t} \qquad x_x = x_0 + V_0 t + \frac{1}{2}g\sin\theta t^2 = \underbrace{\frac{1}{2}g\sin\theta t^2}$

+3

& $[y = y_0]$

← h/sinθ

$U_i = mgz$, what is z

$\sin(\theta) = \frac{h}{l} \qquad l = \frac{h}{\sin\theta}$

$\frac{h}{z} = \frac{h}{\sin\theta(\frac{h}{\sin\theta} - x)}$ $\qquad \frac{h}{\sin\theta} - x \qquad \frac{h}{z} = \frac{h}{h - x\sin\theta} \qquad z = h - x\sin\theta$

$U = mg(h - x\sin\theta)$

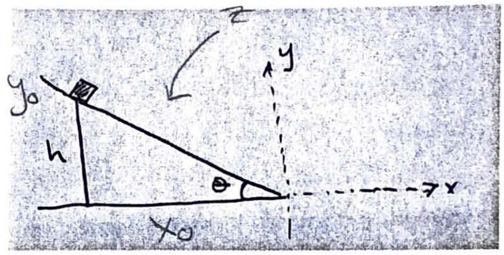
$\left[U = mg(h - (\frac{1}{2}g\sin\theta t^2)\sin\theta)\right]$ ← +3 → $K = \frac{1}{2}mv^2 = \left[\frac{1}{2}m(g^2\sin^2\theta t^2)\right]$

$[mgh - \frac{1}{2}mg^2\sin^2\theta t^2]$

+1

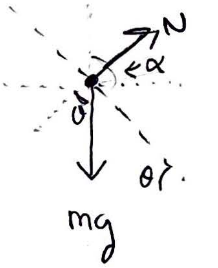
$K + U = mgh$ ← time independent constant ✓

(b) Place the coordinate system so that the y-axis is along the vertical direction, and the origin is at the bottom of the inclined plane. Find the equations of motion. Calculate the kinetic and potential energy of the system as functions of time, and show that their sum is a constant. *(15 points)* +14



$\sin\theta = \frac{h}{z}$ $z = \frac{h}{\sin\theta}$

$\alpha = 90 - \theta$ $X_0 =$



$N = mg\cos\theta$

$\cos\theta = \frac{x_0 \sin\theta}{h}$

$\left[x_0 = \frac{h}{\tan\theta} \right]$

$[y_0 = h]$

$F_x = N\cos\alpha = mg\cos\theta\cos(90-\theta)$

$= mg\cos\theta\sin\theta = ma_x$

$a_x = g\cos\theta\sin\theta$

+4

$F_y = N\sin\alpha - mg = mg\cos^2\theta - mg = ma_y$

$a_y = g\cos^2\theta - g$

$= g(\cos^2\theta - 1)$

$= -g\sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

+3 $\begin{cases} x = x_0 + \frac{1}{2}g\cos\theta\sin\theta t^2 \\ y = y_0 - \frac{1}{2}g\sin^2\theta t^2 \end{cases}$?

$v_x = g\cos\theta\sin\theta t$

$v_y = -g\sin^2\theta t$

$V = gt\sqrt{\cos^2\theta\sin^2\theta + \sin^2\theta\sin^2\theta}$

$gt\sqrt{\sin^2(\cos^2 + \sin^2)}$

$= gt\sin\theta$

$U = mgy = mg(h - \frac{1}{2}g\sin^2\theta t^2)$ +4

$K = \frac{1}{2}mg^2t^2\sin^2\theta$

$U + K = mgh$ +3 ✓

(c) Using the same coordinate system as in (b), find the kinetic energy at the bottom (where the inclined plane meets the ground). +2 *(5 points)*

$$U_i + K_i = U_f + K_f$$

$\cancel{K_i} =$

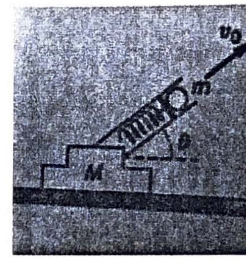
$$E_i = mgh \qquad E_f = \frac{1}{2}mv^2 = K$$

$$E_i = E_f$$

$$K = mgh$$

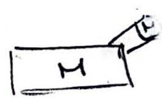
Problem 2 +15 *(25 points)*

A spring gun fires a ball at an angle θ with respect to the horizontal. The mass of the ball is m, and the mass of the spring gun is M. The spring gun is initially at rest, and it is on a frictionless surface. The ball leaves at speed v_0 with respect to the spring gun.



(a) What is the speed v_f at which the spring gun moves, immediately after the ball leaves the muzzle? +8 *(15 points)*

before:



$\leftarrow v.$

let's do conservation at horizontal momentum: $P_{x_i} = 0$

$P_{xf} = m v_0 \cos\theta - M v_f$ ← since speed is +

$P_{xf} = P_{x_i}$

$m v_0 \cos\theta = M v_f$ $\quad \left[v_f = \dfrac{m v_0 \cos\theta}{M} \right]$ ✓

$\cancel{m} v_0 \cos\theta =$

$m(v_0 - v)\cos\theta = M v$

(b) Now, assume that the angle $\theta = 0$. If the spring is originally compressed by a distance x_0 (along the length of the spring), what is the spring gun speed v_f in terms of this distance? +7 *(10 points)*

U of spring ($\frac{1}{2}$ of system) $= \frac{1}{2}Kx_0^2$

+3

now since $\theta = 0$

$$V_f = \frac{mv_0 \cos\theta}{M} = \frac{mv_0}{M}$$ what is V_0?

right when spring is at equilibrium. all energy is transferred to m.

so

+4

$$\cancel{\tfrac{1}{2}}Kx_0^2 = \cancel{\tfrac{1}{2}}mv_0^2$$

$$V_0^2 = \frac{Kx_0^2}{m} \qquad V_0 = x_0\sqrt{\frac{K}{m}}$$

so $$V_f = \frac{m x_0 \sqrt{\frac{K}{m}}}{M}$$

36/45

Problem 3 (45 points)

An object of mass m falls downwards from height h, starting at rest. It is subject to gravity, and to air resistance, which exerts a force F=-ξv, where v is the speed of the object, and ξ is a constant.

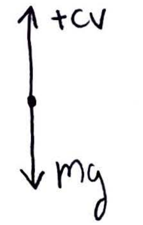
$F = -cv$

$c = \xi$

(a) Find an expression for y(t), which denotes its position in the vertical direction. (20 points)

h

$d = h - y(t)$



$$F = mg - cv = m\ddot{y}$$

$$mg - c\dot{y} = m\ddot{y}$$

$$g - \frac{c}{m}\dot{y} = \ddot{y}$$

$$\cancel{E_f - E_i = W} \quad \cancel{W = \int F dy = \int -cv\,dy = \int -c\dot{y}\,dy}$$

$$\cancel{W = -cy}$$

$$g - \frac{c}{m}v = \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{dv}{g - \frac{c}{m}v}$$

$$t = -\frac{m}{c}\left[\ln\left(g - \frac{c}{m}v\right)\right]_0^v$$

$$= -\frac{m}{c}\left[\ln\left(\frac{g - \frac{c}{m}v}{g}\right) = -\frac{m}{c}\ln\left(1 - \frac{c}{mg}v\right) = t\right.$$

$$\ln\left(1 - \frac{c}{mg}v\right) = -\frac{ct}{m} \qquad e^{-c/m\,t} = 1 - \frac{c}{mg}v$$

$$-\frac{c}{mg}v = e^{-c/m\,t} - 1 \qquad \left(v = \frac{gm}{c}\left(1 - e^{-c/m\,t}\right)\right)$$

$$\frac{gm}{c}\int_0^t \left(1 - e^{-c/m\,t}\right) \to t - \int_0^t e^{-c/m\,t} \to t + \frac{m}{c}\left[e^{-ct/m}\right]_0^t$$

6

+20

$$\frac{gm}{c}\left(t + \frac{m}{c}\left[e^{-\frac{c}{m}t} - 1\right]\right)$$

$$\left\{ y(t) = h - \frac{gmt}{c} - \frac{gm^2}{c^2}\left[e^{-\frac{c}{m}t} - 1\right] \right\}$$

(b) The time at which the object hits the ground, t_f, is given by the equation $y(t_f) = 0$. Find v_f, the velocity of the object when it hits the ground, in terms of this t_f. Simplify this expression as much as possible. *(5 points)*

$y(t) = 0$

$h - \frac{gmt_f}{c} - \frac{gm^2}{c^2}\left[e^{-\frac{c}{m}t_f} - 1\right] = 0$ ← This will solve for t_f

so v_f

$v = \frac{gm}{c}(1 - e^{-\frac{c}{m}t})$ $\left[v_f = \frac{gm}{c}(1 - e^{-\frac{c}{m}t_f})\right]$ ✓

+3

(c) Find the energy that the object dissipates due to air resistance, as it drops from height h to 0. Do this calculation by finding the kinetic and potential energies at starting and final conditions. *(10 points)*

↓ given t_f $\{U_i + K_i = U_f + K_f\}$

$K_i = 0 \rightarrow$ $E_i = mgh$ $E_f = \frac{1}{2} m v_f^2$ ← $U_f = 0$

$\frac{1}{2} m v_f^2 - mgh = W$ ← energy dissipated

$v_f = ?$ $= V(t_f) = \frac{gm}{c}\left(1 - e^{-\frac{c}{m} t_f}\right)$

so

$\frac{1}{2} m \left(\frac{gm}{c}\left(1 - e^{-\frac{c}{m} t_f}\right)\right)^2 - mgh =$ energy dissipated

↓

t_f "solved" in earlier problem

+10

(d) Now, find the energy that the object dissipates due to air resistance, by using the definition $W = \int \vec{F} \cdot d\vec{r}$. (10 points)

$$W = \int F \cdot dr$$

$$= -c \int_0^{t_f} v \, dy$$

$$= -c \left[y \right]_0^{t_f}$$

+ 3

$$= -c \left[y(t_f) - y(0) \right]$$

$$= -c \left[0 - h \right] = ch$$