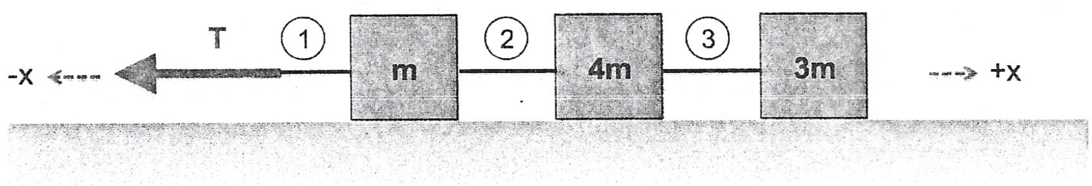
Problem 1 (40 points):

As shown in **Figure 1** below, three blocks of mass *m*, *4m*, and *3m* slide along a frictionless horizontal surface. The blocks are connected by identical strings of negligible mass, labeled 1, 2, and 3. The block-string system is sliding in the *+x* direction at constant speed when, at time $t = 0$, a time-dependent tension force of magnitude *T* is applied in the *-x* direction to string 1. For times $t \geq 0$, the velocity vector of any point on the block-string system is described by the function

$$\vec{v}(t) = (\beta - \alpha t^2)\hat{\imath} ,$$

where α and β are positive constants. For this problem, you can assume that every part of the block-string system moves together and that the strings connecting the blocks are stretched out to their maximum lengths for all times $t \geq 0$.

Figure 1



Part A (10 points): Determine the acceleration vector of any point on the block-string system as a function of time. Express your answer in terms of t, α, and β.

$$\vec{v}(t) = (\beta - \alpha t^2)\hat{i}$$

$$\vec{a}(t) = \vec{v}'(t) = (-2\alpha t)\hat{i}$$

10

Part B (10 points): A specific point of the block-string system is found to pass the origin at time $t = 0$. Determine the position vector of this point as a function of time. Express your answer in terms of t, α, and β.

$$\vec{v}(t) = (\beta - \alpha t^2)\hat{i}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = ~~(\beta t - \frac{\alpha t^3}{3})\hat{i}~~$$

$$\vec{r}(t) = (\beta t - \frac{\alpha t^3}{3} + C)\hat{i}$$

$$0 = \beta(0) - \frac{\alpha(0)}{3} + C$$

$$C = 0$$

$$\vec{r}(t) = (\beta t - \frac{\alpha t^3}{3})\hat{i}$$

10

Part C (10 points): What is the tension magnitude in string 1 at the time when the block-string system is instantaneously at rest? Express your answer in terms of m, α, and β.

$$V = (\beta - \alpha t^2)\hat{i}$$

$$0 = (\beta - \alpha t^2)\hat{i}$$

$$\sqrt{\frac{\beta}{\alpha}} = t$$

$$\vec{a}\left(\sqrt{\tfrac{\beta}{\alpha}}\right) = -2\alpha\left(\sqrt{\tfrac{\beta}{\alpha}}\right)\hat{i}$$

$$T = F_{Net} = (m_1 + m_2 + m_3) - 2\alpha\sqrt{\tfrac{\beta}{\alpha}}$$

$$T = \cancel{8m}\ -16m\left(\alpha\sqrt{\tfrac{\beta}{\alpha}}\right)$$

↑ sign −1

9

Part D (10 points): What is the tension magnitude in string 3 at the time when the block-string system is instantaneously at rest? Express your answer in terms of m, α, and β.

$$t = 7\sqrt{\frac{\beta}{\alpha}}$$

$$a = -2\alpha\, 7\sqrt{\frac{\beta}{\alpha}}$$

$$T = m_3(a)$$

$$T = -6m\left(\alpha 7\sqrt{\frac{\beta}{\alpha}}\right)$$

9

Problem 2 (40 points):

A daredevil wants to design a catapult that will launch him from one incline to another, as shown in **Figure 2** below. Both inclines are identical, having a base length L and an angle of $\alpha_0 = 30°$. The horizontal distance between the inclines is $3L$. The daredevil initially stands on a platform at the bottom of the incline on the left. The combined mass of the daredevil and platform is M_1. The platform is connected via a rope and pulley to a vertically hanging counterweight of mass M_2. When the counterweight is released from rest, it descends and pulls the platform and daredevil up the incline. At the very top of the incline, the platform is stopped suddenly, launching the daredevil. The acceleration magnitude due to gravity is g.

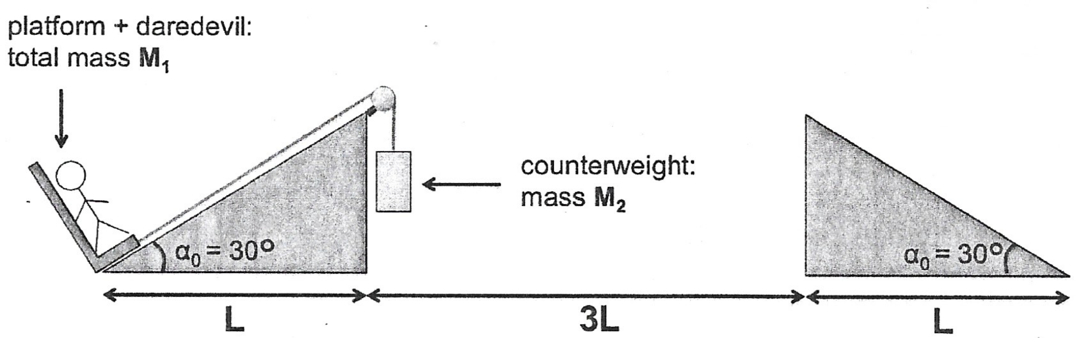
For this problem, you can assume the following:

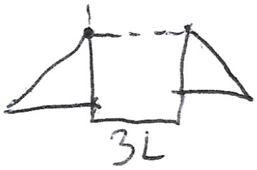
- The daredevil is launched from the very top of the incline at the same speed that the platform attains just before it is stopped suddenly.
- The size of the platform is negligible with respect to the size of the incline, such that the distance that the platform travels can be approximated as the entire length of the incline and the height from which the daredevil is launched can be approximated as the height of the incline.
- The friction between the platform and incline is negligible.
- The mass of the rope and pulley and the friction of the pulley are negligible.
- Air resistance is negligible.
- The counterweight doesn't hit the ground before the daredevil is launched (e.g., it can keep descending into a hole in the ground).

You may find the following relations helpful:

$$\sin 30^\circ = \tfrac{1}{2} \quad \cos 30^\circ = \tfrac{\sqrt{3}}{2} \quad \sin 60^\circ = \tfrac{\sqrt{3}}{2} \quad \cos 60^\circ = \tfrac{1}{2}$$

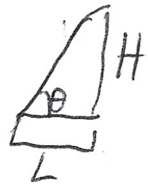
Figure 2





~~tan θ = 1~~

$g = -9.8$ $\theta = 30°$



$\tan(\theta) = \frac{H}{L}$

$L\tan\theta = H$

~~$\tan(\frac{H}{L}) = \theta$~~

Part A (10 points): ~~What~~ is the speed with which the daredevil must be launched from the top of the incline on the left side in order to just reach the top of the incline on the right side? Express your answer in terms of L and g.

$x = V_0 \cos\theta \, t$

$t = \frac{x}{V_0\cos\theta} = \frac{3L}{V_0\cos\theta}$ +2

$y = \cancel{L\tan\theta} + V_0 \sin\theta t + \frac{1}{2}gt^2$

$0 = V_0 \sin\theta\left(\frac{3L}{V_0\cos\theta}\right) + \frac{1}{2}g\left(\frac{3L}{V_0\cos\theta}\right)$ +2

$-3L\tan\theta = \frac{1}{2}g\left(\frac{9L^2}{V_0^2\cos^2\theta}\right)$

$\frac{-6L\tan\theta}{g} = \frac{9L^2}{V_0^2\cos^2\theta}$

$\frac{-2\left(\frac{\sin\theta}{\cos\theta}\right)}{3gL} = \frac{1}{V_0^2\cos^2\theta}$

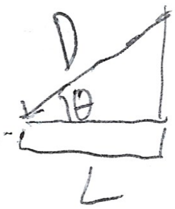
$\frac{-2(\sin\theta\cos\theta)}{3gL} = \frac{1}{V_0^2}$ +3

$\sqrt{\frac{3gL}{-\left(\frac{\sqrt{3}}{2}\right)}} = V_0$ +2

10

$g = -9.8$

Part B (10 points): What acceleration magnitude must the platform have as it ascends the incline in order to attain the speed of Part A? Express your answer in terms of *g*.



$$V_f^2 = V_i^2 + 2a(D)$$

$$\frac{3gL}{-\frac{\sqrt{3}}{2}} = 0 + 2a\left(\frac{L}{\frac{\sqrt{3}}{2}}\right)$$

$$\cos\theta = \frac{L}{D}$$

$$D = \frac{L}{\cos\theta}$$

$$\frac{3gL}{-\frac{\sqrt{3}}{2}} = 2a\left(\frac{L}{\frac{\sqrt{3}}{2}}\right)$$

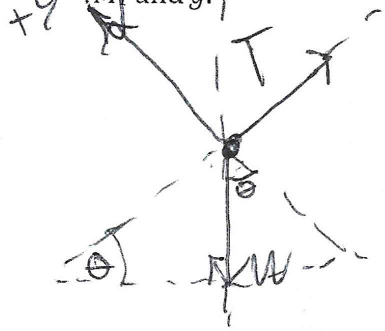
$$-\frac{6gL}{\sqrt{3}} = \frac{4aL}{\sqrt{3}}$$

$$-\frac{3}{2}(g) = a$$

(10)

$g = -9.8$

Part C (10 points): What tension magnitude is required in the rope in order to achieve the acceleration magnitude from Part B? Express your answer in terms of M_1 and g.



$$F_{Net_y} = 0 = N - W\cos\theta$$

$$N = W\cos\theta$$

$$= m_1 g \frac{\sqrt{3}}{2}$$

+2

$$F_{Net_x} = T - W\sin\theta = T - \frac{mg}{2} = m_1 a$$

+3

$$T + \frac{m_1 g}{2} = M_1\left(-\frac{3}{2}g\right)$$

because $g < 0$

$$T = M_1 g\left(-\frac{3}{2} + \frac{1}{2}\right)$$ +3

$$T = M_1 g(-1)$$

$$T = -M_1 g$$ +1

9

$g = -9.8$

Part D (10 points): What counterweight mass M_2 is required in order to achieve the acceleration magnitude from Part B? Express your answer in terms of M_1.

T from here has to equal the T pulling our daredevil up. Also, the acceleration of both the daredevil & platform combo should equal the acceleration of this weight ✓

$$F_{Net} = T - W = M_2 a$$

$$-M_1 g - M_2 g = M_2 a$$

$$-M_1 g = M_2 (a + g)$$

$$-M_1 g = M_2 (-\tfrac{3}{2} g + g)$$

$$-M_1 g = M_2 (-\tfrac{1}{2} g)$$

$$2M_1 = M_2$$

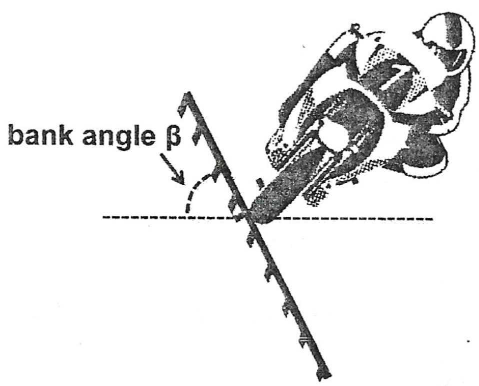
10

$\beta = \theta$

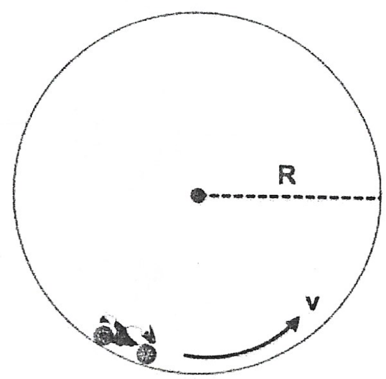
Problem 3 (20 points):

As shown in **Figure 3**, a motorcyclist rides at constant speed on the inside of a cylindrical wall banked at an angle β with respect to the horizontal. The radius of the motorcycle's circular path is R and the coefficient of static friction between the motorcycle's tires and the wall is μ_s. The acceleration magnitude due to gravity is g. For this problem, the sizes of the motorcyclist and motorcycle can be assumed negligible with respect to the radius of the cylindrical wall.

Figure 3



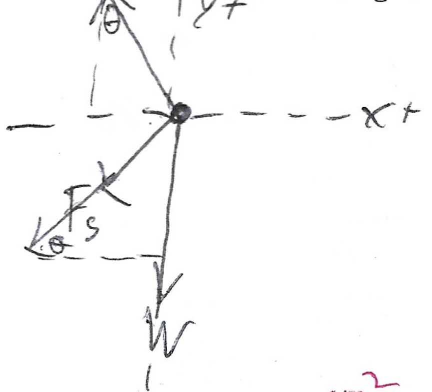
view from side



view from above

$\beta = \theta$

Part A (10 points): What is the fastest speed that the motorcyclist can maintain without skidding? Express your answer in terms of R, μ_s, g, and β.



+1 $F_{Net_y} = 0 = N\cos\theta - W$

+ f1124p1

$\frac{mg}{\cos\theta} = N$

$F_{Net_x} = F_s\cos\theta + N\sin\theta = \frac{m v_{max}^2}{R}$ +2

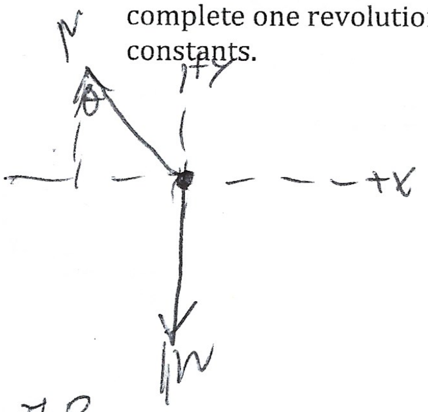
+2 $\mu_s\left(\frac{mg}{\cos\theta}\right)\cos\theta + \left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m v_{max}^2}{R}$

$\mu_s mg + mg\tan\theta = \frac{m v_{max}^2}{R}$

+2 $mg(\mu_s + \tan\theta) = \frac{m v_{max}^2}{R}$

$\sqrt{Rg(\mu_s + \tan\beta)} = v_{max}$

Part B (10 points): Now assume that the banked wall shown in **Figure 3** becomes extremely icy, lowering the coefficient of static friction to zero. In order to avoid skidding up or down the wall, what amount of time must it take the motorcyclist to complete one revolution? Express your answer in terms of R, g, β, and fundamental constants.



$$v = \frac{2\pi R}{T}$$

$$Net_y = 0 = N\cos\theta - W$$

$$N = \frac{mg}{\cos\theta}$$

$$F_{Netx} = N\sin\theta = \frac{mv^2}{R}$$

$$mg\tan\theta = \frac{m(4\pi^2 R^2)}{RT^2}$$

$$g\tan\theta = \frac{4\pi^2 R}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 R}{g\tan\beta}}$$

$$T = 2\pi\sqrt{\frac{R}{g\tan\beta}}$$ +10