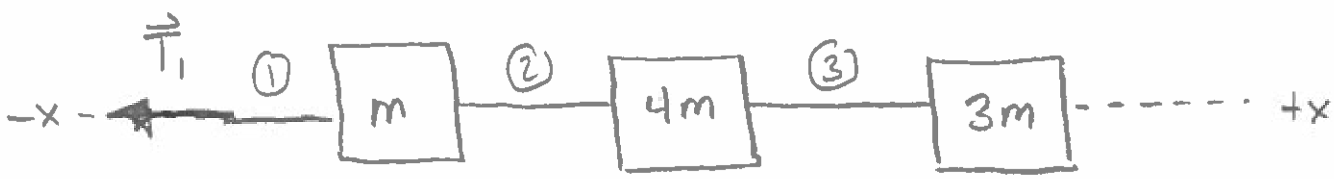
PHYSICS 1A - LECTURE 1
MIDTERM #1 SOLUTIONS

Problem #1 (40 points)



$$\vec{v}(t) = (\beta - \alpha t^2)\,\hat{\imath}, \qquad \alpha, \beta > 0$$

Part A (10 points):

- acceleration: $\vec{a}(t) = \dfrac{d\vec{v}(t)}{dt} = (-2\alpha t)\,\hat{\imath}$

$$\boxed{\vec{a}(t) = (-2\alpha t)\,\hat{\imath}}$$

Part B (10 points):

- position: $$x(t_2) - x(t_1) = \int_{t_1}^{t_2} v_x(t)\,dt$$

- Choose $t_1 = 0$, such that $x(t_1) = x(0) = 0$:

$$x(t_2) = \int_0^{t_2} v_x(t)\,dt = \int_0^{t_2} (\beta - \alpha t^2)\,dt = \left. \beta t - \frac{\alpha t^3}{3} \right]_0^{t_2}$$

$$x(t_2) = \beta t_2 - \frac{\alpha t_2^3}{3}$$

- replace $t_2 \to t$ and write position vector as $\vec{r}(t) = x(t)\hat{\imath}$:

$$\boxed{\vec{r}(t) = \left(\beta t - \frac{\alpha t^3}{3}\right)\hat{\imath}}$$

Part C (10 points):

- System at rest when $\vec{v}(t) = (\beta - \alpha t^2)\hat{\imath} = 0 \rightarrow t_{rest} = \pm\sqrt{\frac{\beta}{\alpha}}$

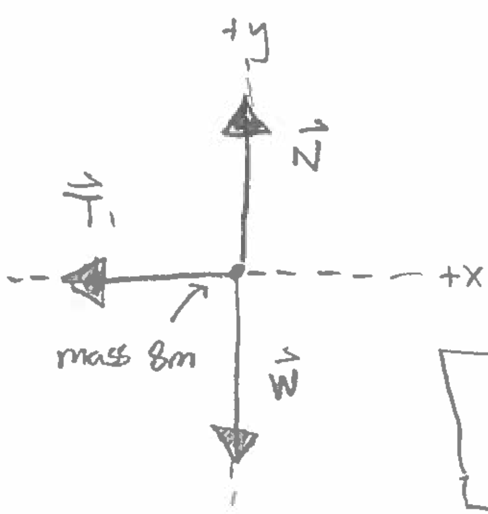
keep only "+" solution since expression for $\vec{v}(t)$ only valid for $t \geq 0$

- acceleration at this time is:

$$\vec{a}(t) = (-2\alpha t)\hat{\imath}, \quad t_{rest} = +\sqrt{\frac{\beta}{\alpha}}$$

$$\vec{a}(t_{rest}) = \left(-2\alpha \cdot \sqrt{\frac{\beta}{\alpha}}\right)\hat{\imath} = -2\sqrt{\alpha\beta}\,\hat{\imath} \qquad a_x = -2\sqrt{\alpha\beta}$$

- free-body diagram for composite object of mass $(m + 4m + 3m)$ being pulled by string ①:



Newton's 2nd Law:

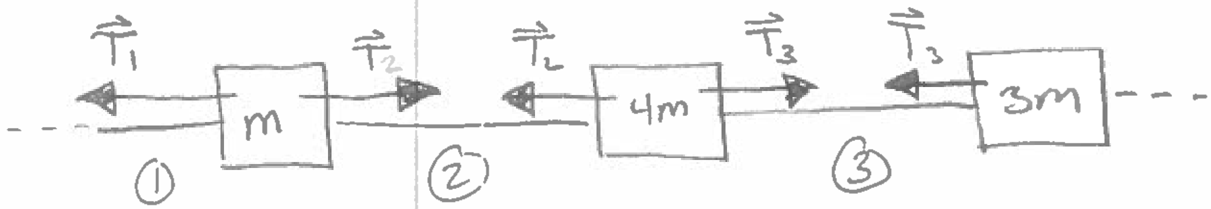
$$F_{net,x} = (8m)a_x \rightarrow -|\vec{T}_1| = (8m)(-2\sqrt{\alpha\beta})$$

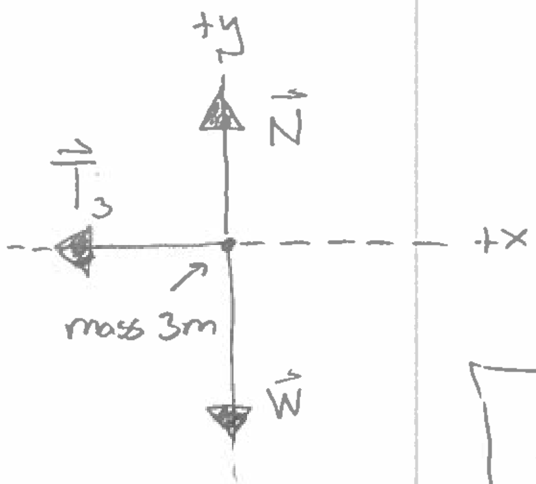
$$F_{net,y} = (8m)a_y \rightarrow +|\vec{N}| - |\vec{W}| = 0$$

$$\boxed{\text{tension in string ①: } |\vec{T}_1| = 16m\sqrt{\alpha\beta}}$$

Part D (10 points):

- Free-body diagram for mass 3m being pulled by string 3:





Newton's 2^{nd} Law:

$$F_{net,x} = (3m)a_x \rightarrow -|\vec{T}_3| = (3m)(-2\sqrt{\alpha\beta})$$

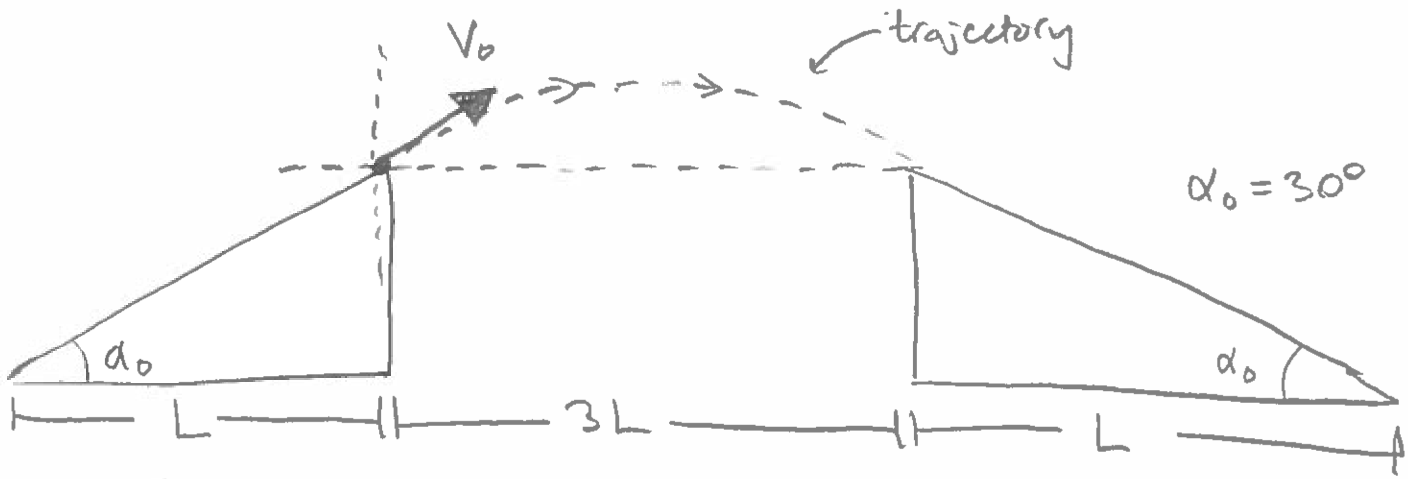
$$F_{net,y} = (3m)a_y \rightarrow |\vec{N}| - |\vec{W}| = 0$$

$$\boxed{\text{tension in string 3: } |\vec{T}_3| = 6m\sqrt{\alpha\beta}}$$

Problem #2 (40 points)

Part A (10 points):

- Between the inclines, the daredevil is in projectile motion. The starting and ending height of the trajectory is the same since the inclines are identical. Thus, the range formula can be used:



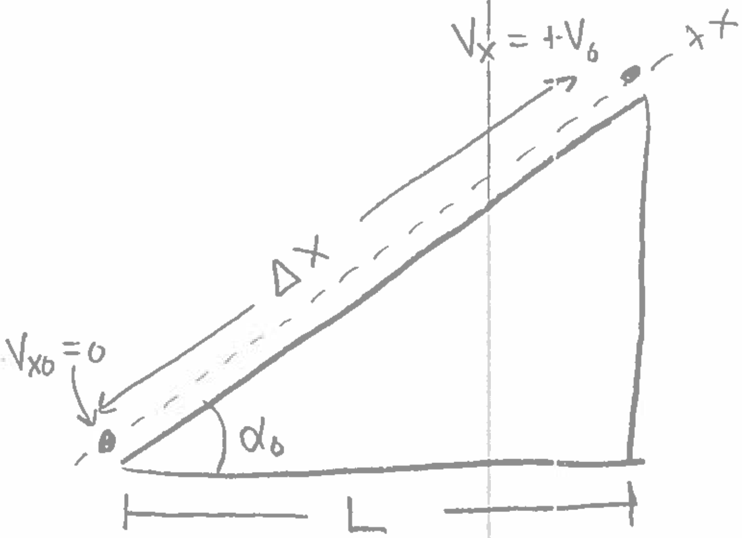
- range $= 3L = \dfrac{V_0^2 \sin(2\alpha_0)}{g}$

- using $\sin(2\alpha_0) = \sin(60°) = \dfrac{\sqrt{3}}{2}$, $\quad 3L = \dfrac{\sqrt{3}}{2} \cdot \dfrac{V_0^2}{g}$

$$V_0^2 = \frac{6Lg}{\sqrt{3}} \longrightarrow \boxed{\text{launch speed: } V_0 = \sqrt{(2\sqrt{3})Lg}}$$

Part B (10 points):

- When the counterweight is released, the system has constant acceleration and we can apply the constant acceleration kinematic equations:



- $V_{x0} = 0$
- $V_x = +V_0 = +\sqrt{(2\sqrt{3})Lg}$
- Δx: $(\Delta x)\cos\alpha_0 = L$

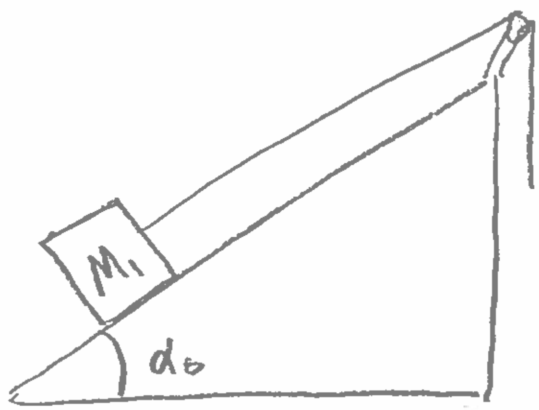
$$\Delta x = \frac{L}{\cos\alpha_0} = \frac{L}{\left(\frac{\sqrt{3}}{2}\right)} = \left(\frac{2\sqrt{3}}{3}\right)L$$

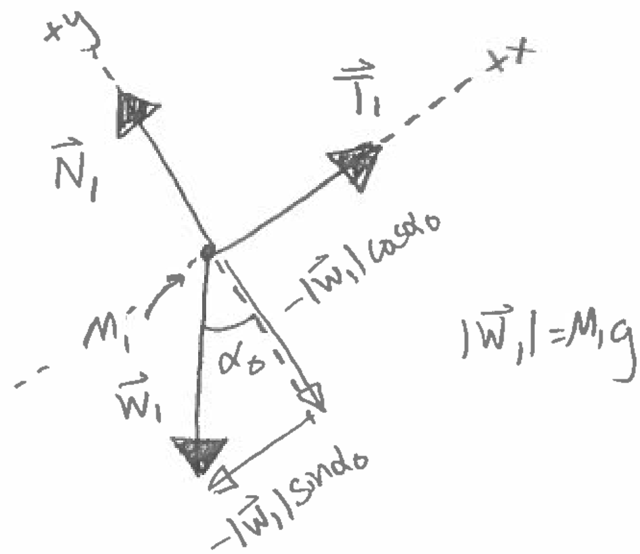
$$V_x^2 = V_{x0}^2 + 2a_x\Delta x \rightarrow \cancel{2\sqrt{3}L}g = 0 + 2a_x\left(\cancel{\frac{2\sqrt{3}}{3}L}\right)$$

$$\boxed{\text{The acceleration magnitude is: } |a_x| = \left(\frac{3}{2}\right)g}$$

Part C (10 points):

- Free-body diagram for daredevil + platform:





- Newton's 2^{nd} Law:

$$F_{net,x} = M_1 a_x \rightarrow +|\vec{T}_1| - M_1 g \overbrace{\sin\alpha_0}^{=\frac{1}{2}} = M_1 \overbrace{a_x}^{=+\frac{3}{2}g}$$

$$F_{net,y} = M_1 a_y \rightarrow +|\vec{N}_1| - M_1 g \cos\alpha_0 = 0$$

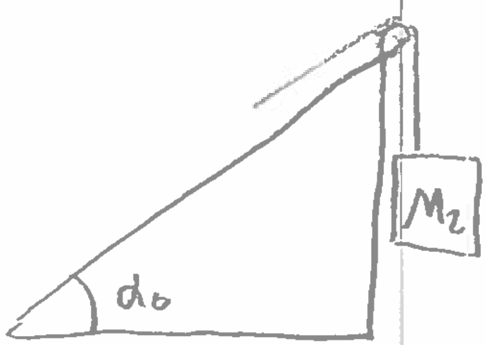
- The x-equation gives:

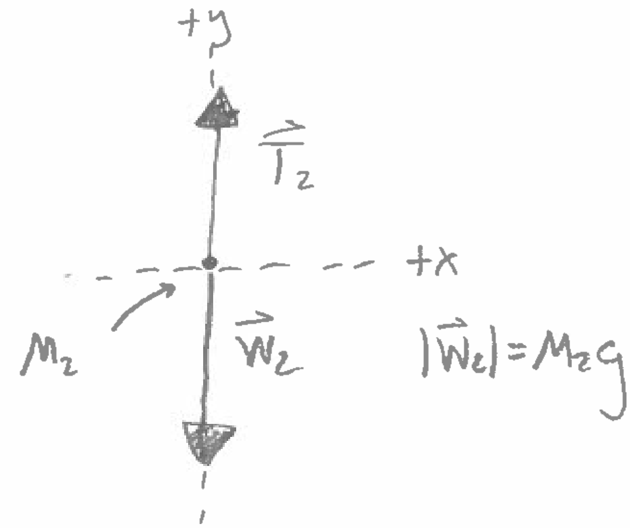
$$+|\vec{T}_1| - \frac{1}{2} M_1 g = +\frac{3}{2} M_1 g \rightarrow |\vec{T}_1| = 2M_1 g$$

- The tension magnitude is: $\boxed{|\vec{T}_1| = 2M_1 g}$

Part D (10 points):

- Free-body diagram for counterweight:





- Newton's 2^{nd} Law:

$$F_{net,x} = M_2 a_x \longrightarrow 0 = 0$$
$$F_{net,y} = M_2 a_y \longrightarrow |\vec{T}_2| - M_2 g = M_2 a_y$$

- massless rope + massless frictionless pulley:

$$|\vec{T}_2| = |\vec{T}_1| = 2M_1 g$$

- blocks connected together:

$$\underbrace{a_y = -a_x}_{\text{when } M_1 \text{ moves up, } M_2 \text{ moves down}} = -\frac{3}{2} g$$

- The y-equation gives:

$$2M_1\cancel{g} - M_2\cancel{g} = M_2\left(-\frac{3}{2}\cancel{g}\right)$$

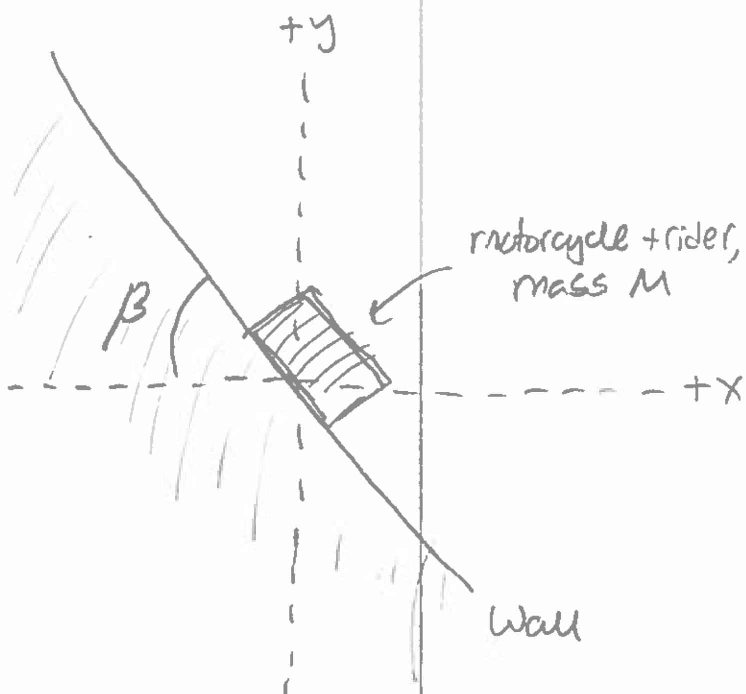
$$2M_1 = M_2\left(1 - \frac{3}{2}\right) = -\frac{1}{2}M_2$$

$$\boxed{M_2 = -4M_1}$$

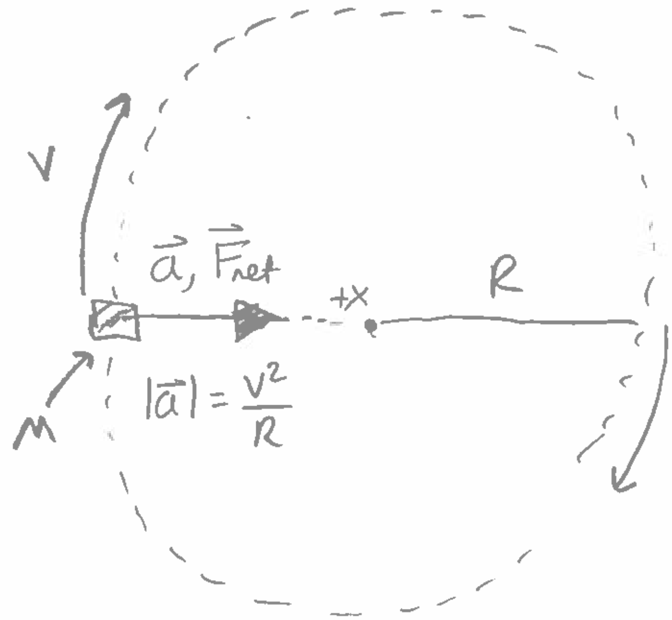
This is a <u>non-physical answer</u> – it is impossible to use this setup to launch the daredevil by the desired distance!

Problem #3 (20 points)

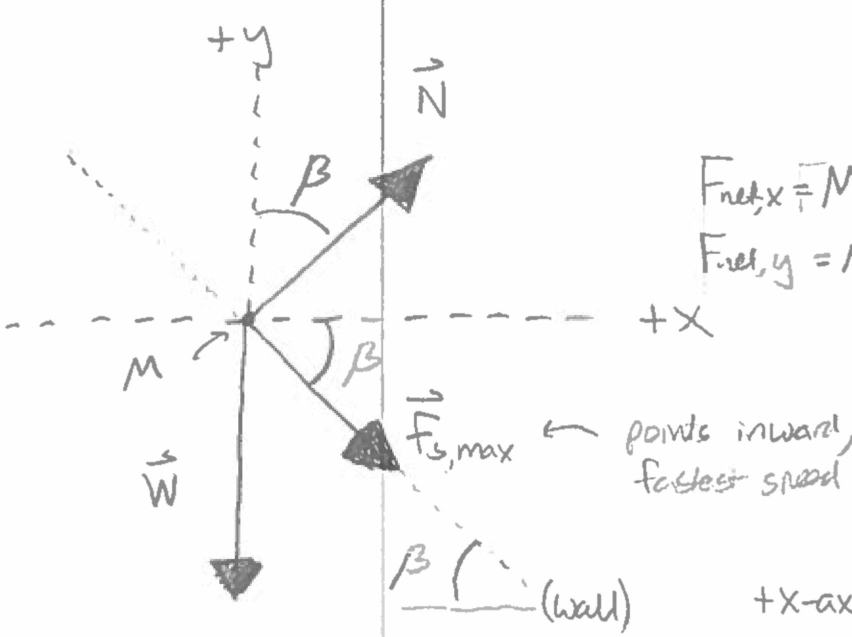
Part A (10 points):



view from side



view from above



Newton's 2nd Law:

$$F_{net,x} = Ma_x \rightarrow |\vec{N}| \sin\beta + |\vec{f}_{s,max}| \cos\beta = \frac{MV^2}{R}$$

$$F_{net,y} = Ma_y \rightarrow |\vec{N}| \cos\beta - |\vec{f}_{s,max}| \sin\beta - Mg = 0$$

$$|\vec{f}_{s,max}| = \mu_s |\vec{N}|$$

+x-axis points towards circle center

- The x-equation becomes:

$$|\vec{N}| \sin\beta + \mu_s |\vec{N}| \cos\beta = \frac{MV^2}{R}$$

- The y-equation becomes:

$$|\vec{N}| \cos\beta - \mu_s |\vec{N}| \sin\beta - Mg = 0$$

- Solving y-equation for $|\vec{N}|$ and substituting into x-equation:

$$|\vec{N}| \left[\cos\beta - \mu_s \sin\beta\right] = Mg$$

$$|\vec{N}| = \frac{Mg}{\cos\beta - \mu_s \sin\beta}$$

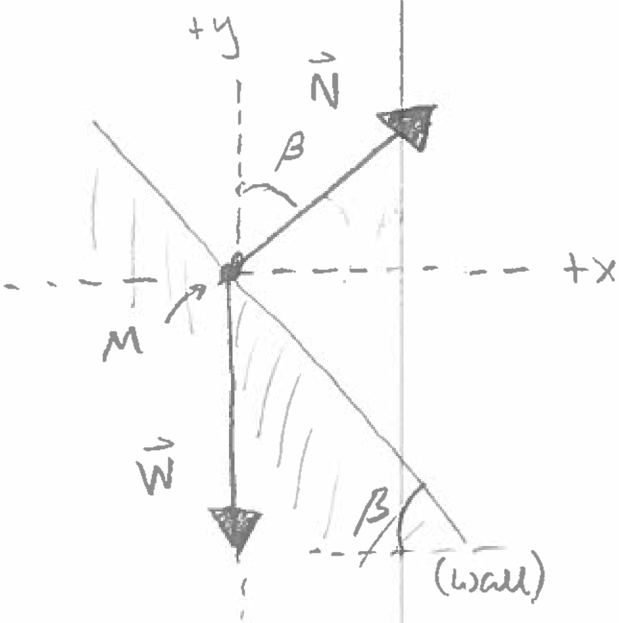
$$\left(\frac{\cancel{M}g}{\cos\beta - \mu_s \sin\beta}\right) \sin\beta + \mu_s \left(\frac{\cancel{M}g}{\cos\beta - \mu_s \sin\beta}\right) \cos\beta = \frac{\cancel{M}V^2}{R}$$

$$g \cdot \left[\frac{\sin\beta + \mu_s \cos\beta}{\cos\beta - \mu_s \sin\beta}\right] = \frac{V^2}{R}$$

- Solving for V: $\boxed{V_{max} = \sqrt{Rg \left[\frac{\sin\beta + \mu_s \cos\beta}{\cos\beta - \mu_s \sin\beta}\right]}}$

Part B (10 points):

- This problem is equivalent to Part A only with static friction set to zero:



$$F_{net,x} = Ma_x \rightarrow |\vec{N}|\sin\beta = \frac{MV^2}{R} = M\left(\frac{4\pi^2R}{T^2}\right)$$

$$F_{net,y} = Ma_y \rightarrow |\vec{N}|\cos\beta - Mg = 0$$

$$V = \frac{2\pi R}{T} \rightarrow \frac{MV^2}{R} = M\left(\frac{4\pi^2 R}{T^2}\right)$$

- The y-equation gives: $|\vec{N}|\cos\beta = Mg \rightarrow |\vec{N}| = \dfrac{Mg}{\cos\beta}$

- Substitution into x-equation gives:

$$\left(\frac{\cancel{M}g}{\cos\beta}\right)\sin\beta = \cancel{M}\left(\frac{4\pi^2R}{T^2}\right)$$

$$g\tan\beta = \frac{4\pi^2 R}{T^2}$$

- Solving for T:

$$T^2 = \frac{4\pi^2 R}{g\tan\beta}$$

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}\left(\frac{1}{\tan\beta}\right)} = 2\pi\sqrt{\frac{R}{g}\cot\beta}}$$