MECHANICAL & AEROSPACE ENGINEERING DEPARTMENT
UNIVERSITY OF CALIFORNIA, LOS ANGELES

MAE 182A

MATHEMATICS OF ENGINEERING

WINTER, 2010

INSTRUCTOR

CHATTERJEE, A. K.

FINAL EXAMINATION

March 17, 2010

INSTRUCTIONS: SHOW ALL CALCULATIONS ON THESE PAGES.
ATTACH ADDITIONAL PAGES AS NECESSARY.

NAME ____

Your Student ID ________________________________

Problem 1 (5+5+10)

Problem 2 (6+6+8)

1. Describe and derive the formula for the each of the following methods in solving general second order linear differential. Also, mention the kind of problem where these methods can be applied.
   (a) method of variation of parameters
   (b) Method of reduction of order
2. If $y(t) = 1/t$ is one of the fundamental solution of the differential equation

$$t^2\ddot{y} + 3t\dot{y} + y = 0,\ t > 0$$

derive the other fundamental solution using the reduction of order.

Problem 3 (5+5+10+5)

a) Determine the radius of convergence of the following power series:

$$\sum_{n=0}^{\infty} 2^n x^n$$

b) Determine and classify the singular points of the following differential equation:

$$2x(x-2)^2 y'' + 3xy' + (x-2)y = 0$$

For the following differential equation

$$x^2 y'' + xy' + (x-2)y = 0$$

c) determine the indicial equation, its roots, and the recurrence relation
d) find the series solution (x>0) corresponding to the larger root

Problem 4 (5+6+6)

Problem 5 (15+10)

a. Find the fundamental matrix $\Phi(t)$ satisfying $\Phi(0) = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ for the following homogeneous differential equation

$$\mathbf{x}' = \mathbf{A}\mathbf{x} \text{ where } \mathbf{A} = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$$

b. Using the above result, solve the nonhomogeneous differential equation

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$$

Do not integrate.

Problem 6 **(5+10+10)**

Consider the heat conduction problem of finding the temperature $u(x,t)$ in a uniform rod of length ℓ with insulated ends so that there is no passage of heat through them. The initial temperature distribution is given by $u(x,0) = f(x)$ where $f(x)$ is a known function of time in $0 \le x \le \ell$.

a. Write down the mathematical formulation of the problem with the governing differential equation for $u(x,t)$, and all boundary and initial conditions needed for the complete solution of $u(x,t)$.

b. Using a variable separable solution $u(x,t) = X(x)T(t)$, reduce the problem to a Sturm-Liouville boundary value problem. Show that $\lambda = 0$ is an eigen value and find the corresponding eigen function.

c. Derive the complete solution of the problem in terms of $f(x)$.