

a) If pressures are equal at B and C, they must lie on a const. pressure paraboloid surface as sketched in the figure. If  $z_B = 0$ , then from Eq. 2.32:

$$z_C = 0.3 \text{ m} = \frac{\Omega^2 r_C^2}{2g} = \frac{\Omega^2 (0.3 \text{ m})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})}$$

Rearrange for  $\Omega$ :

$$\Omega = 8.09 \frac{\text{rad}}{\text{s}} = 77 \frac{\text{rev}}{\text{min}}$$

b) The minimum pressure occurs where the highest paraboloid pressure contour is tangent to BC, as shown in the figure. Again using Eq. 2.32 with  $\Omega$  (in  $\frac{\text{rad}}{\text{s}}$ ) from part (a):

$$z = \frac{\Omega^2 r^2}{2g} = \text{const.} = r \tan(45^\circ)$$

Plug in for  $\Omega$ ,  $g$ , and  $\tan(45^\circ)$ :

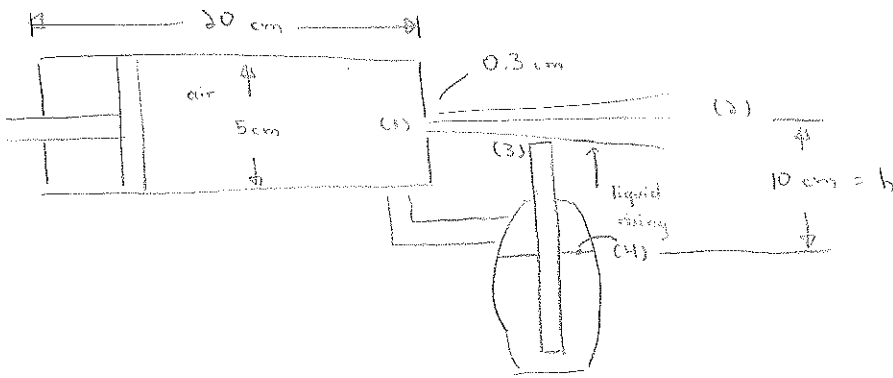
$$\text{const.} + 3.333r^2 - r = 0 \equiv f(r) \text{ for any constant pressure line}$$

The minimum is found when  $\frac{df}{dr} = 0$ , so we get:

$$r = 0.15 \text{ m}$$

Since  $z = r \tan(45^\circ)$ :

$$z = 0.15 \text{ m}$$



Assumptions:

- 1) Flows are steady, incompressible, and inviscid.
- 2) Air is an ideal gas.
- 3) The reservoir is open to the atmosphere.
- 4) Device is horizontal.
- 5) Water velocity through the tube is low.

Analysis:

Point 1 is at the pump exit, 2 is far from the exit on a horizontal line, point (3) is at the exit of the liquid tube but close enough to point 1 that they can be assumed coincident, and point 4 is at the free surface of the reservoir liquid. Thus,  $P_2 = P_4 = P_{atm}$  and  $P_1 = P_3$ . Also,  $z_1 = z_2 = z_3 = 0$  and  $z_4 = -h$ , and  $v_3 = v_3 = v_4 = 0$ .

Apply B. eqn. to water (3-4):

$$\frac{P_3}{\rho g} + \frac{v_3^2}{2g} + z_3 = \frac{P_4}{\rho g} + \frac{v_4^2}{2g} + z_4 \rightarrow \frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} - h \rightarrow P_1 - P_{atm} = -\rho_{water} g h \quad (1)$$

Apply B. eqn. to air (1-2):

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \rightarrow \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_{atm}}{\rho g} \rightarrow v_1 = \sqrt{\frac{2(P_{atm} - P_1)}{\rho_{air}}} \quad (2)$$

$$\text{where } \rho_{air} = \frac{P}{RT} = \frac{95 \text{ kPa}}{\left(0.287 \frac{\text{kPa} \cdot \text{m}^3}{\text{kg} \cdot \text{K}}\right) (293 \text{ K})} \rightarrow \rho_{air} = 1.13 \frac{\text{kg}}{\text{m}^3}$$

Combining (1) and (2):

$$v_1 = \sqrt{\frac{2\rho_{water} g h}{\rho_{air}}} = \sqrt{\frac{2(1000)(9.81 \frac{\text{m}}{\text{s}^2})(0.1 \text{ m})}{1.13}} \rightarrow v_1 = 41.7 \frac{\text{m}}{\text{s}}$$

2. Since the flow is steady and incompressible:

$$v_1 A_1 = v_{\text{piston}} A_{\text{piston}}$$

$$v_{\text{piston}} = \left( \frac{A_1}{A_{\text{piston}}} \right) v_1 = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_{\text{piston}}^2} v_1 = \frac{(0.3 \text{ cm})^2}{(5 \text{ cm})^2} (41.7 \frac{\text{m}}{\text{s}}) \rightarrow \boxed{v_{\text{piston}} = 0.15 \frac{\text{m}}{\text{s}}}$$