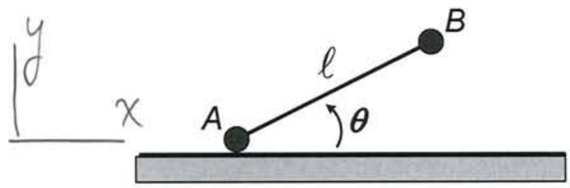
MAE 102 MIDTERM EXAM 2 FALL 2007

1. Two small balls, each of mass m, are connected by a massless rigid rod of length ℓ. Ball A slides without friction on the horizontal floor. The system is released from rest with $\theta = 90^\circ$, and ball B falls to the right. Find the velocity of ball A and the angular velocity of the rod when $\theta = 30^\circ$.

Give answers in terms of m, g and ℓ.



$$V = mg\ell\sin\theta$$

$$y_G = \frac{\ell}{2}\sin\theta \qquad v_{Gy} = \dot{y}_G = \dot{\theta}\frac{\ell}{2}\cos\theta$$

$$\Sigma F_x = 0 \Rightarrow a_{Gx} = 0 + v_{Gx} = 0$$

$$T = \frac{1}{2}(2m)v_G^2 + \frac{1}{2}(2m)\left(\dot{\theta}\frac{\ell}{2}\right)^2 = \frac{m\ell^2\dot{\theta}^2}{4}(1+\cos^2\theta)$$

$$T + V = mg\ell \Rightarrow \dot{\theta}^2 = \frac{4g(1-\sin\theta)}{\ell(1+\cos^2\theta)}$$

$$\theta = 30^\circ \Rightarrow \boxed{\dot{\theta} = -2\sqrt{\frac{2}{7}\frac{g}{\ell}}}$$

$$\bar{v}_A = \bar{v}_G + \underbrace{\dot{\theta}\frac{\ell}{2}(\sin\theta\,\bar{i} - \cos\theta\,\bar{j})}_{\bar{v}_{A/G}} = \boxed{-\sqrt{\frac{1}{14}g\ell}\;\bar{i}}$$

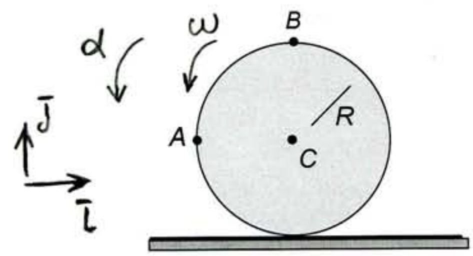
Alternatively:

$$\bar{v}_B = \bar{v}_A + \bar{v}_{B/A} = v_A\bar{i} + \dot{\theta}\ell\left(-\frac{1}{2}\sin\theta\,\bar{i} + \cos\theta\,\bar{j}\right)$$

$$m\bar{v}_A + m\bar{v}_B = 0 \Rightarrow v_A = \dot{\theta}\frac{\ell}{2}\sin\theta, \quad \bar{v}_B = \dot{\theta}\ell\left(-\frac{1}{2}\sin\theta\,\bar{i} + \cos\theta\,\bar{j}\right)$$

$$T = \frac{m}{2}\left(v_A^2 + v_B^2\right) = \frac{m\ell^2\dot{\theta}^2}{4}(1+\cos^2\theta)$$

2. The disc of radius $R = 1\,\mathrm{m}$ rolls without slipping. Point C has velocity v_C to the right, and v_C is *decreasing.* In the position shown, the magnitude of the velocity vector of point A is $4\,\mathrm{m/s}$, and the magnitude of the acceleration vector of point A is $8\,\mathrm{m/s^2}$. Find the velocity and acceleration vectors of point B.



$$\bar{v}_c = -\omega R \bar{i} = v_c \bar{i} \qquad \bar{a}_c = -\alpha R \bar{i}$$

$$\dot{v}_c < 0 \Rightarrow \alpha > 0$$

$$\bar{v}_A = \bar{v}_c + \bar{v}_{A/c} = \bar{v}_c - \omega R \bar{j} \Rightarrow v_A^2 = 2\omega^2 R^2 = (4\,m/s)^2$$

$$v_c > 0 \Rightarrow \underline{\omega = -2\sqrt{2}\ m/s}$$

$$\bar{v}_B = \bar{v}_c + \bar{v}_{B/c} = \bar{v}_c - \omega R \bar{i} = -2\omega R \bar{i} = \boxed{4\sqrt{2}\ m/s\ \bar{i}}$$

$$\bar{a}_A = \bar{a}_c + \bar{a}_{A/c} = \bar{a}_A + \omega^2 R \bar{i} - \alpha R \bar{j} = (\omega^2 - \alpha) R \bar{i} - \alpha R \bar{j}$$

$$a_A^2 = (\omega^4 - 2\omega^2 \alpha + 2\alpha^2) R^2 = (8\,m/s^2)^2$$

$$\Rightarrow \alpha = 0, \quad \underline{\alpha = 8}$$

$$\bar{a}_B = \bar{a}_c + \bar{a}_{B/c} = \bar{a}_c - \alpha R \bar{i} - \omega^2 R \bar{j} = -2\alpha R \bar{i} - \omega^2 R \bar{j}$$

$$\boxed{\bar{a}_B = -16\,m/s^2\,\bar{i} - 8\,m/s^2\,\bar{j}}$$

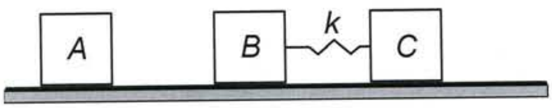
3. The three blocks slide without friction on the horizontal floor, and each block has mass m. Initially, block A has velocity v_0 to the right, blocks B and C are at rest, and the spring is unstretched. On impact, blocks A and B stick together. The spring does not compress during the impact. Find

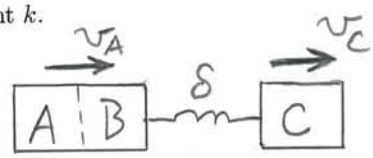
   (a) the velocity of blocks A and B immediately after the impact,

   (b) the maximum compression of the spring,

   (c) the maximum velocity of block C.

   Give answers in terms of v_0, m, and the spring constant k.





(a) $m v_0 = 2m v_A \Rightarrow \underline{\underline{v_A = v_0/2}}$

① $2m v_A + m v_C = m v_0$

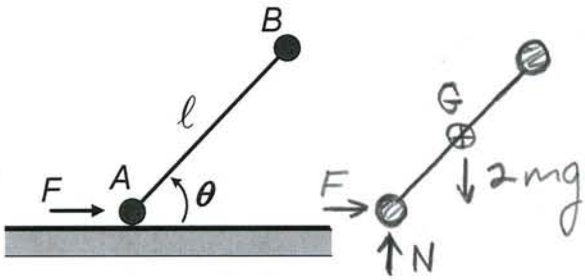
② $\frac{2m}{2} v_A^2 + \frac{m}{2} v_C^2 + \frac{k}{2}\delta^2 = \frac{2m}{2}\left(\frac{v_0}{2}\right)^2$

(b) $\underline{v_A = v_C}$ ① + ② $\Rightarrow \delta = \pm v_0\sqrt{\frac{m}{6k}}$

max compression $= \underline{\underline{v_0\sqrt{\frac{m}{6k}}}}$

(c) $a_C = 0 \Leftrightarrow \underline{\delta = 0}$ ① + ② $\Rightarrow \underline{\underline{v_C = \frac{2}{3} v_0}}$

4. Two small balls, each of mass m, are connected by a massless rigid rod of length ℓ. Ball A slides without friction on the horizontal floor, and a constant horizontal force F acts on ball A. Initially, the system is at rest.



$$a_{Gy} = \ddot{\theta}\frac{\ell}{2}\cos\theta$$

$$\bar{H}_G = \frac{m\ell^2}{2}\dot{\theta}\,\bar{k}$$

$$\dot{\bar{H}}_G = \frac{m\ell^2}{2}\ddot{\theta}\,\bar{k}$$

(a) If θ remains constant with $\cos\theta = 3/5$, the value of F is

i. mg

$\cos\theta = 4/5$ $\boxed{\text{ii. } \frac{8}{3}mg}$

$\cos\theta = 3/5$ iii. $\frac{3}{2}mg$

iv. $\frac{3}{4}mg$

v. $\frac{5}{3}mg$

$$N - 2mg = 2m\,a_{Gy} = m\ell\ddot{\theta}\cos\theta \quad ①$$

$$F\frac{\ell}{2}\sin\theta - N\frac{\ell}{2}\cos\theta = \dot{H}_G = \frac{m\ell^2}{2}\ddot{\theta} \quad ②$$

$\underline{\ddot{\theta} = 0}$ ① + ② $\Rightarrow F, N$

(b) If $F = 3mg$, the horizontal component of the acceleration of the center of mass is

$F = 3mg$ i. $\frac{3}{2}g$

ii. $\frac{1}{3}g$

iii. $\frac{3}{4}g$

iv. $\frac{4}{3}g$

$F = 5mg$ $\boxed{\text{v. } \frac{5}{2}g}$

$$F = 2m\,a_{Gx}$$

$$a_{Gx} = \frac{F}{2m}$$

(c) If $F = 3mg$, the angular acceleration of the rod for the initial position is

i. $\frac{g}{\ell} \cdot \frac{5\sin\theta - 3\cos\theta}{1 + \cos^2\theta}$

$F = 3mg$ ii. $\frac{g}{\ell} \cdot \frac{3\sin\theta - 2\cos\theta}{1 + \cos^2\theta}$

iii. $\frac{g}{\ell} \cdot \frac{3\cos\theta - 2\sin\theta}{1 + \cos^2\theta}$

$F = 5mg$ $\boxed{\text{iv. } \frac{g}{\ell} \cdot \frac{5\sin\theta - 2\cos\theta}{1 + \cos^2\theta}}$

v. $\frac{g}{\ell} \cdot \frac{5\cos\theta - 2\sin\theta}{1 + \cos^2\theta}$

① + ② $\Rightarrow \ddot{\theta}, N$