SOLUTION

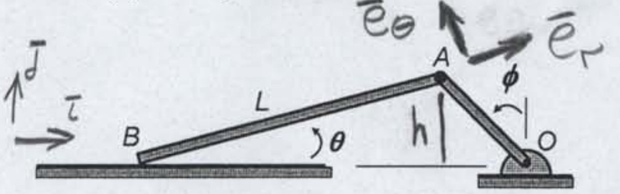
MAE 102 MIDTERM EXAM 1 FALL 2007

$\Rightarrow \ddot{\phi} = 0$

1. Rod AO rotates at the constant rate $\dot{\phi} = 6$ rad/s, and end B of rod AB slides on the horizontal floor. The length of rod AO is $5L/13$. For the position in which $\phi = 0$, find:

   (a) the angular velocity $\dot{\theta}$ of rod AB and the velocity of B,

   (b) the angular acceleration $\ddot{\theta}$ of rod AB and the acceleration of B.



$\phi = 0$: $\cos\theta = \frac{12}{13}$ $\sin\theta = 5/13$

$\bar{e}_\theta = \underbrace{-\frac{5}{13}\bar{I} + \frac{12}{13}\bar{J}}$

$\phi = 0$: $\bar{v}_A = \bar{v}_B + \bar{v}_{A/B} \Rightarrow -\frac{5}{13}L\dot{\phi}\bar{I} = v_B\bar{I} + L\dot{\theta}\left(-\frac{5}{13}\bar{I} + \frac{12}{13}\bar{J}\right) \Rightarrow$

$\underline{\dot{\theta} = 0}$ $\underline{\bar{v}_B = -\frac{5}{13}L\dot{\phi} = -\frac{30}{13}L/s}$

$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B} \Rightarrow -\frac{5}{13}L\dot{\phi}^2\bar{J} = a_B\bar{I} + L\ddot{\theta}\underbrace{\left(-5/13\,\bar{I} + \frac{12}{13}\bar{J}\right)}_{\bar{e}_\theta} \Rightarrow$

$\underline{\ddot{\theta} = -\frac{5}{12}\dot{\phi}^2 = -15\ \text{rad}/s^2}$ $\underline{\bar{a}_B = \frac{5}{13}L\ddot{\theta} = -\frac{75}{13}L/s^2}$

OR: $h = L\sin\theta = \frac{5}{13}L\cos\phi \Rightarrow \dot{\theta}\cos\theta = -\frac{5}{13}\dot{\phi}\sin\phi \xrightarrow{\phi=0} \underline{\dot{\theta} = 0}$

$\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta = -\frac{5}{13}\left(\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi\right) \xrightarrow{\phi=0} \underline{\ddot{\theta} = -\frac{5}{12}\dot{\phi}^2}$

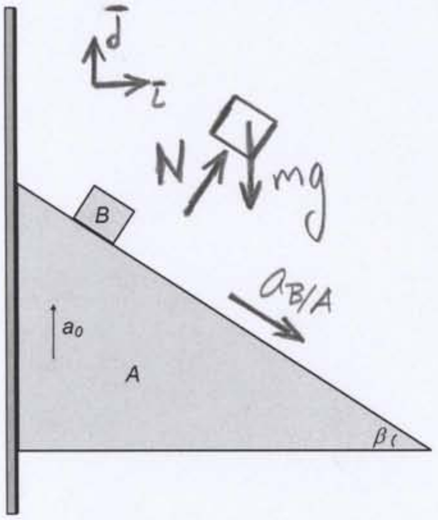
$x_B \xrightarrow{+}$ from O: $x_B = -\left(L\cos\theta + \frac{5}{13}L\sin\phi\right)$

$v_B = \dot{x}_B = L\dot{\theta}\sin\theta - \frac{5}{13}L\dot{\phi}\cos\phi \xrightarrow{\phi=0} \underline{v_B = -\frac{5}{13}L\dot{\phi}}$

$\bar{a}_B = \ddot{x}_B = L\left[\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right] + \frac{5}{13}L\left[-\ddot{\phi}\cos\phi + \dot{\phi}^2\sin\phi\right]$

$\xrightarrow{\phi=0}$ $\underline{\bar{a}_B = -\frac{25}{12(13)}\dot{\phi}^2} = \underline{-\frac{75}{13}L/s^2}$

2. Block A is given the constant vertical acceleration a_0, and block B slides without friction on block A. Find a_0 so that the absolute acceleration of block B is horizontal. Find the magnitude of the absolute acceleration of block B and the magnitude of the acceleration of block B relative to block A.



$$\bar{a}_B = a_0\bar{j} + a_{B/A}(\cos\beta\,\bar{i} - \sin\beta\,\bar{j})$$

$$\bar{a}_B = \underbrace{a_{B/A}\cos\beta}_{a_{Bx}}\,\bar{i} + \underbrace{(a_0 - a_{B/A}\sin\beta)}_{a_{By}}\,\bar{j}$$

$$\Sigma F_x = ma_x: \quad N\sin\beta = m\,a_{B/A}\cos\beta$$

$$\Sigma F_y = ma_y: \quad N\cos\beta - mg = m(a_0 - a_{B/A}\sin\beta)$$

$$a_{By} = a_0 - a_{B/A}\sin\beta = 0$$

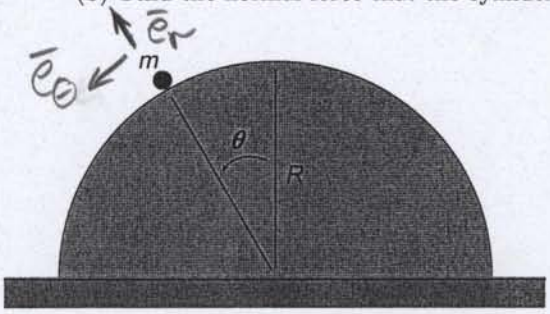
$$\Rightarrow N = mg/\cos\beta \quad \underline{a_0 = g\tan^2\beta} \quad \underline{a_{B/A} = g\tan\beta/\cos\beta}$$

$$\underline{a_B = a_{Bx} = g\tan\beta}$$

3. Mass m slides without friction on the fixed cylinder of radius R. The mass is released from rest with $\theta = 0$.

(a) Find the $\dot{\theta}$ as a function of θ.

(b) Find the normal force that the cylinder exerts on the mass as a function of θ.

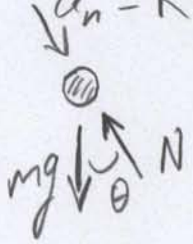


(a) $V = mgR\cos\theta \qquad T = \frac{1}{2}m(R\dot{\theta})^2$

$$T + V = \frac{mR^2}{2}\dot{\theta}^2 + mgR\cos\theta = V(0) = mgR$$

$$\Rightarrow \dot{\theta}^2 = \frac{2g}{R}(1-\cos\theta) \Rightarrow \underline{\underline{\dot{\theta} = \sqrt{\frac{2g}{R}(1-\cos\theta)}}}$$

(b) $a_n = R\dot{\theta}^2$



$$\bar{e}_r: \Sigma F = N - mg\cos\theta = -mR\dot{\theta}^2$$

$$\Rightarrow \underline{\underline{N = mg(3\cos\theta - 2)}}$$

4. The block of mass m is released from rest when the spring is unstretched.

(a) If there is no friction, the maximum stretch in the spring is

i. $\frac{mg}{k}\sin\beta$

ii. $\frac{2mg}{k}\sin\beta$

iii. $\frac{3mg}{2k}\sin\beta$

iv. $\frac{2mg}{3k}\sin\beta$

v. $\frac{mg}{2k}\sin\beta$

$T_1 + V_1 = T_2 + V_2$:

$0 = \frac{1}{2}kx^2 - mgx\sin\beta$

$\Rightarrow x = 0,\ x = \frac{2mg}{k}\sin\beta$

(b) If the static and kinetic coefficients of friction between the block and the inclined plane are both 0.25 $\tan\beta$, the maximum stretch in the spring is

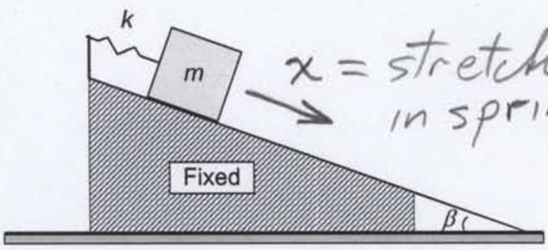
i. $\frac{mg}{k}\sin\beta$

ii. $\frac{2mg}{k}\sin\beta$

iii. $\frac{3mg}{2k}\sin\beta$ $\mu = .25\tan\beta$

iv. $\frac{2mg}{3k}\sin\beta$

v. $\frac{mg}{2k}\sin\beta$ $\mu = .75\tan\beta$



x = stretch in spring

$V_g = -mgx\sin\beta$

$V_s = \frac{1}{2}kx^2$

$T = \frac{1}{2}m\dot{x}^2$

$N = mg\cos\beta \quad f = \mu N = \mu mg\cos\beta$

$U_{1-2} = -\frac{1}{2}kx^2 + mgx\sin\beta - \mu mgx\cos\beta = T_2 - T_1 = 0$

$\Rightarrow x = \frac{2mg}{k}(\sin\beta - \mu\cos\beta)$

(a) $\mu = 0 \Rightarrow \frac{2mg}{k}\sin\beta$ (b) $\mu = \frac{\tan\beta}{4}$ or $\frac{3\tan\beta}{4}$