Version

MATH 61: INTRODUCTION TO DISCRETE STRUCTURES MIDTERM #1

INSTRUCTOR: SPENCER UNGER

Name: Solutions	
ID #	
Section	
Good Luck! Be sure to justify your answers!	
No calculators, books or notes are allowed	

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
Total	75	

Be careful, there are problems on both sides of the paper!

(1) (15 points) Prove by induction that for all $n \ge 4$, $2^n \ge n^2$.

Base Case
$$N=4$$

$$2^{4} = 16 \quad \text{and} \quad 4^{2} = 16$$

$$50 \quad 2^{4} \geq 4^{2}$$

Inductive Step: Assume that 2" = n2 for Some N = 4.

$$(n+1)^2 = n^2 + 2n+1 \le n^2 + n^2 \le 2^n + 2^n = 2 \cdot 2^n = 2^{n+1}$$

This uses the idential hypothesis.

Were were using that $n^2 \ge 2n+1$ when $n \ge 4$.

- (3) (15 points; each of (a), (b) and (c) are weighted equally) Answer the following questions.
 - (a) Let $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and $C = \{b, c, 2, 3\}$. Determine each of the following. If it is a set, then write the set by listing the elements. If it is a number, then write which one. If it is a statement, then write true or false.

(i)
$$|B \cup C|$$

(ii)
$$A \times B$$

$$\left\{ (\alpha, 1), (\alpha, 2), (\alpha, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3) \right\}$$
(iii) $(A \cup B) - C$

(iv)
$$C \cap (B \times B)$$

$$(v) (A \cap B) \subseteq C$$

(b) Let A_1 and A_2 be sets. Find disjoint sets B_1 and B_2 such that $A_1 \cup A_2 = B_1 \cup B_2$.

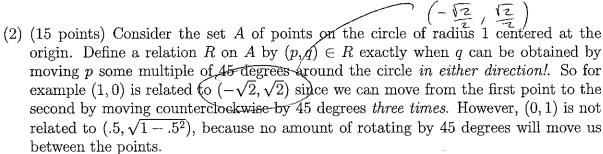
Set
$$B_1 = A_2 \times A_1 - A_2$$

 $B_2 = A_2$

(c) Prove or disprove the following statement. For any sets A, B and X, if $A \subseteq B \subseteq X$, then $X - A \subseteq X - B$.

This is false. Take
$$A = \{a\}$$
 $B = \{a,b\}$ $X = \{a,b,c\}$
Clearly $A \subseteq B \subseteq X$, but

$$X - A = \{b,c\} \notin \{e\} = X - B$$



(a) Show that R is an equivalence relation.

Reflorive

Symmetric

For any points p,q eA, assume (p,q) eR.

So there is a whole number k so that moving 45° k times open in one direction goes from p to q. To see that (q,p) eA move 45° k times in the opposite direction.

Transitive For any points piger & If (pig) & and (qir) & R

then we get a multiple of 45° for each say les and ke.

To see (pir) & R more peither (kither) × 45° or |ki-ki|×45°

either abockuse or counter abochemise based on the direction between part qual qual r.

(b) List the elements of [(1,0)].

$$(1,0)$$
, $(\overline{\Sigma}_{2},\overline{\Sigma}_{2})$, $(0,1)$, $(-\overline{\Sigma}_{2},\overline{\Sigma}_{2})$, $(-1,0)$, $(-\overline{\Sigma}_{2},-\overline{\Sigma}_{2})$, $(0,-1)$, $(\overline{\Sigma}_{2},-\overline{\Sigma}_{2})$

(4) (15 points) Define the following sets: (a) Let X be the set of strings with alphabet $\{0, 1\}$. (b) Let Y be the set of strings with alphabet $\{a, b\}$. (c) Let Z be the set of strings with alphabet $\{0, 1, a, b\}$. Define a function $F: X \times Y \to Z$ by $F(\alpha, \beta) = \alpha \frown \beta$. Recall that $\alpha \frown \beta$ is the concatenation of α and β . (a) Is F injective? Justify your answer. If F(x,B) = F(x,8) then 2 B = y=8 So the length of a must be the length of of and similarly for B and 8, since the alphabets don't overlap. Now we get X= y and B=8 since the corresponding entries must be equal. (b) Is F surjective? Justify your answer. there is an a to the left of a l.

No. The string at is not in the range since So it can't be in the form X P where acx and BeY.

(c) Recall that the range of F is the set $\{z \in Z \mid \text{there is } (x,y) \in X \times Y \text{ such that } \}$ F(x,y)=z. Show that $X \cup Y$ is a subset of the range of F.

Let I & denote the empty String Naif $x \in X$ $F(\alpha, \lambda) = \alpha^{\gamma} \lambda = \alpha$ IF BEY F(X,B) = X P = P So XUY is contained in the range of F.

