1. Consider the sets S, T, and U defined below:

$$S = \{n \in \mathbb{Z} : \text{there is } m \in \mathbb{Z} \text{ such that } n = m^2\} = \{0, 1, 4, 9, \ldots\}$$
$$T = \{r \in \mathbb{R} : \text{there is } x \in \mathbb{R} \text{ such that } r = \sqrt{x}\} = [0, \infty)$$
$$U = \{r \in \mathbb{R} : -1 < r < 1\} = (-1, 1)$$

Which of the following statements are true about these sets?

Circle all correct answers.

a. S and U are disjoint.

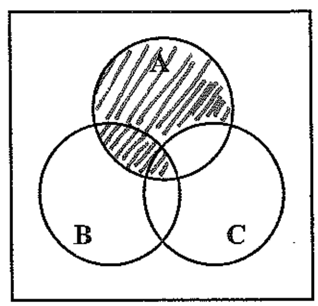
b. $T \cap U = U$

c. $S \subseteq T$

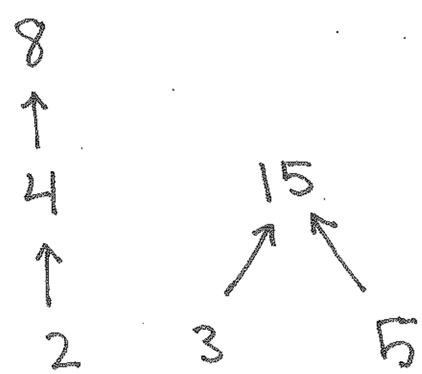
d. $S \cap T = T$

e. $T \cup U = \{r \in \mathbb{R} : r + 1 > 0\} = \{r \in \mathbb{R} : r > -1\} = (-1, \infty)$

2. In the Venn diagram below, shade the region corresponding to $(A \cap B) \cup (\overline{C} \cap A)$.



3. Draw a Hasse diagram for the partial order of *divisibility* on the set $S = \{2, 3, 4, 5, 8, 15\}$.



4. For a.–c., if the function f is *injective*, circle it.

   If the function f is not *injective* then in the blank spaces below that function, write down two elements a, b of the domain such that $a \neq b$ but $f(a) = f(b)$.

   a. $f : [0, \infty) \to \mathbb{R}$ given by $f(x) = (x-1)^2 + 1$.

   0 ______ 2 ______

   b. $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(n) = \begin{cases} -n^2 & \text{if } n < 0 \\ n & \text{if } n \geq 0 \end{cases}$

   ______ ______

   c. $f : \mathcal{P}(\{1,2,3\}) \to \mathcal{P}(\{1,2,3\}) : S \mapsto \{1,2,3\} - S$.

   ______ ______

5. State the *inclusion–exclusion principle*: If A and B are finite sets, then...

   $|A \cup B| = |A| + |B| - |A \cap B|$

6. Let $S = \{0,1,2,3\} \times \{1,2,3\}$ and let $\sim$ be the relation on S defined by

$$(a,b) \sim (c,d) \quad \text{if} \quad a+b = c+d$$

   List all the elements of the $\sim$-equivalence class $[(1,1)]$.
   $\sim$ is an equivalence relation (you do not need to prove this)

   $[(1,1)] = \{(0,2), (1,1)\}$

7. One way to represent a relation is using a table.

Let $S = \{A, B, C, D\}$. We define a relation $\sqsubset$ on S by writing an X in the <u>row</u> for X and the <u>column</u> for Y in the table if (and only if) $X \sqsubset Y$:

	A	B	C	D
A	X	X		
B	X		X	
C		X	X	
D				X

For example, reading the first row: $A \sqsubset A$ and $A \sqsubset B$, but $A \not\sqsubset C$ and $A \not\sqsubset D$.

Using the table above, answer the following questions about the relation $\sqsubset$.

In a.–c. you do not have to justify "Yes."

a. Is the relation $\sqsubset$ *reflexive*? If you answered "No," why not?

No. $B \not\sqsubset B$.

b. Is the relation $\sqsubset$ *symmetric*? If you answered "No," why not?

Yes.

c. Is the relation $\sqsubset$ *transitive*? If you answered "No," why not?

No. $A \sqsubset B$ and $B \sqsubset C$ but $A \not\sqsubset C$.

8. Prove the following statement by *mathematical induction*: For all integers $n \geq 1$,

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

Try to keep your proof as organized as you possibly can!

Base case: $n=1$

$$\sum_{k=1}^{1} k(k+1) = 1\cdot 2 = 2$$

and $\frac{1(1+1)(1+2)}{3} = \frac{1\cdot 2\cdot 3}{3} = 2$ ✓.

Induction step: Assume $P(n)$.

$$\sum_{k=1}^{n+1} k(k+1) = \sum_{k=1}^{n} k(k+1) + (n+1)\cdot(n+2)$$

$$= \frac{n(n+1)(n+2)}{3} + (n+1)(n+2) \quad \text{by } P(n).$$

$$= \left[\frac{n}{3}+1\right](n+1)(n+2)$$

$$= \left[\frac{n+3}{3}\right]\cdot(n+1)(n+2) = \frac{(n+1)(n+2)(n+3)}{3} \checkmark$$

So $P(n+1)$. ✓
