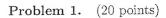
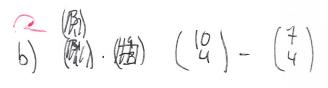
2



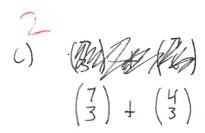
Compute the number of 4-subsets A of $\{1,2,\ldots,10\}$ such that:

- a) A has no even numbers,
- b) A has at least one number ≤ 3 ,
- c) the smallest number in A is divisible by 3.
- d) the sum of numbers in A is exactly 11.











MIDTERM 1 (MATH 61, FALL 2012)

Problem 2. (20 points)

Find closed formulas for the following sequences :

- a) $a_1 = 1$, $a_{n+1} = a_n \cdot \binom{n+1}{2}$
- b) $a_1 = 1$, $a_2 = 1$, $a_{n+1} = a_{n-1} a_n$ for $n \ge 2$.

Note: you can express a_n in terms of Fibonacci numbers F_n .

an =?

b)
$$u_{n+1} = u_{n-1} - u_n$$

$$t^{n+1} = t^{n-1} - t^n$$

$$t^2 = t + t + t - t$$

$$t^2 + t - t = 0$$

$$t^2 + t - t = 0$$

$$t = -\frac{t}{2}$$

$$t = -\frac{t}{2}$$

4/20

Problem 3. (15 points)

Let $a_n = 111 \cdots 1$ (n ones). Suppose a_k is divisible by 97. Use induction to show that $a_{k \cdot n} = 0 \mod 97$, for all $n \ge 1$.

いこし home = 0 mod 17

Induction Step: # Show their if Ukin = D mad 47, then UK(m1) = 0 may 97.

16 431 U314

Induction Step rewritten:

ak(n+1) If \(\(\mathbb{l}\) \(\mathbb{l}\) \

((k.(nh) = [1]...| ((nk) + [1]...|)

Mones

Th + 111...11.10(n-1)k Omas 97, induction

USsymption assuluption

= 0 mod 97, all terms in ak.(111) = 0 mod 97.

Problem 4. (15 points)

There are 6 candidates in a student election, to form a 3-student committee. The university printed bulletins with the names of all 6 candidates, and every student must mark exactly 3 of them. The total number of students is 105. Prove that:

- a) at least one candidate has 53 votes,
- b) at least 6 students cast identical bulletins.
- (1) Assume all Landidates recein less than 53 votes. Euch Student Lasts 3 votes, 105 students, 315 total votes Votes.

315 > 53, Contrudiction.

Thus Ut teas a one Undident received mon than 53 votes

D). Assume less than 6 cast identical ballots.
Total ballots, (6) options

105 20 75, Contradiction. Problem 5. (30 points, 2 points each) TRUE or FALSE?

2945 - 26-4= OM3

x2-x2-0

Circle correct answer with ink. No explanation required.

- T (1) The number of functions from $\{A, B, C, D, F\}$ to $\{1, 2, 3\}$ is equal to 15.
- \bigcirc F (2) The relation R on integers is defined by $x \to_R y$ if and only if $x^2 y^2 = 0$. Then R is an equivalence relation.
- \bigcirc F (3) The relation R on integers is defined by $x \to_R y$ if and only if $2x + y = 0 \mod 3$. Then R is an equivalence relation.
- \bigcirc F (4) Fibonacci numbers $F_n < (1.99)^n$ for all $n \ge 1$.
- (T) F (5) There are infinitely many Fibonacci numbers which are divisible by 3.
- \mathbf{T} (6) The number of permutations of $\{1, 2, 3, 4, 5\}$ is smaller than 101.
- T (7) The number of 3-permutations of $\{1, 2, 3, 4, 5, 6\}$ is equal to $\binom{6}{3}$.
- \bigcirc F (8) Sequence $a_n = 1 + 3/n^2$, $n \ge 1$, is decreasing.
- T (9) The number of permutations of $\{1, 2, ..., n\}$ which end with 1 is equal to n!
- **T** $\widehat{\mathbb{F}}$ (10) For every $A, B \subset \{1, 2, \dots, 12\}$ we have $|A \cap B| < |A \cup B|$.
- F (11) $a_{n+1} = a_{n-1} + 2a_{n-3} 4a_{n-5}$ is a linear homogeneous recurrence relation.
- **T** $\widehat{\mathbf{F}}$ (12) $a_{n+1} = a_n \cdot a_{n-1}$ is a linear homogeneous recurrence relation.
- T (13) The number of anagrams of MISSISSIPPI which begin with M is greater than the number of anagrams which begin with S.
- T (F) (14) The Inclusion-Exclusion Principle for three sets is the following formula:

$$|A\cup B\cup C|\,=\,|A|+|B|+|C|-|A\cap B|-|A\cap C|-|B\cap C|$$

 ${f \hat{T}}$ F (15) The following parabolas are drawn in the plane:

$$y = x^2 - n^2 x - n^3$$
, $n = 1, ..., 12$.

Then the regions of the plane separated by these parabolas can be colored with two colors in such a way that no two adjacent regions have the same color.