Instructions:

- You have from Friday 20 November 2020 at 00.00am to 11.59pm Pacific Time to solve this exam.
- Scan your solutions and upload them to Gradescope by Friday 20 November at 11.59pm Pacific Time. You should submit readable scans, and not pictures of your solutions. Please make sure to match the problems on the exam template with the respective parts in your solutions.
- This exam is open book, and you are allowed to use the textbook, and all resources from the lecture, or similar resources.
- You are not allowed to ask for help from thirds, nor give help to others taking this exam. Students suspected of academic dishonesty may be reported to the Dean of Students. This leads to a process which could end in suspension or dismissal.

Code of honor

Academic integrity is of the uttermost importance. By taking part in this evaluation, you are accepting the following code of honor:

I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.

Problem 1. (5 points) Can a graph have exactly five vertices of degree 1? Either draw an example of a graph satisfying this property or explain why such a graph can't exist.

Problem 2. (5 points) Let $x, y, w, z \in \mathbb{N}$ be such that $x \geq 0$, $y \geq 1$, $w > 0$ and $z \geq 3$. Using the stars and bars method, find the number of solutions of the following equation:

$$x + y + w + z = 20\,.$$

Problem 3. (5 points)
Let $X = \{0, 1, 2\}$, and let $G = (V, E)$ be the bipartite graph with vertex set $V = V_1 \cup V_2$, where $V_1 = X$ and $V_2 = \mathcal{P}(X)$. For any $v_1 \in V_1$ and $v_2 \in V_2$ there is an edge $\{v_1, v_2\} \in E$ if and only if v_1 is an element of v_2. Answer the following, by providing brief explanations:

1. Is G connected?
2. Suppose $v_1 \in V_1$. What is the degree of v_1?
3. Suppose $v_2 \in V_2$. What is the degree of v_2?

Problem 4. (5 points) Consider the recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} + c_4 a_{n-4}$$

for $n \geq 4$. Show that if r is a root of the polynomial

$$x^4 - c_1 x^3 - c_2 x^2 - c_3 x - c_4,$$

then $a_n = r^n$ is a solution to the given recurrence relation.

Math 61 Midterm 2

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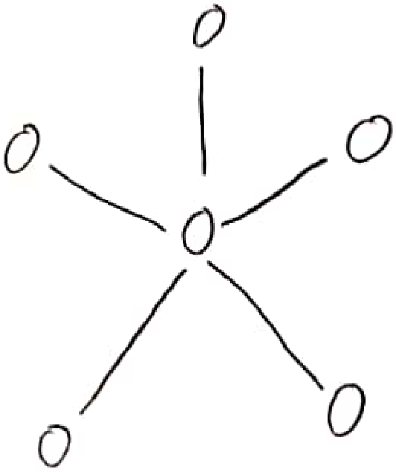
1. This question can have 2 interpretations:
if it means "Can a graph of any number of vertices contain 5 vertices of degree 1," then the answer is 'yes.' an example would be:



In this graph, there are 5 vertices with exactly 1 edge going towards a central

If it means, "Can a graph exist composed of exactly 5 vertices each having a degree of 1," then the answer is "no." In such a system, the sum of all degrees of all the vertices would be 5. Each edge by definition add 1 to the degree of 2 vertices. This means that the total sum of the degree of all vertices is equal to 2 · the number of edges. This implies that for all possible graphs, the sum of all degrees is even. The graph in question has a degree sum of 5, Which is not-even, so it is not possible.

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2. let $x, y, w, z \in \mathbb{N}$ $x \geq 0$ $y \geq 1$ $w > 0$ $z \geq 3$
find number of solutions to

$$x + y + w + z = 20$$

the stars and bars method is most useful when all inputs have a minimum value of 0

let $y' = y - 1 \Rightarrow y' \geq 0$
$z' = z - 3 \Rightarrow z' \geq 0$

$$\Rightarrow x + y' + w + z' = 20 - 3 - 1 = 16$$

The number of solutions to this equation is equal to the number of solutions of the original equation.
Consider the 16 on the right-side to represent an amount of stars broken into 4 groups by 3 bars. Create 16 spaces to be filled by the stars and bars. In this situation, it is known that there are $\binom{16+3}{3}$ ways to choose the location of bars, with repetition. From here, there is only one way to distribute the stars.

$\therefore$ the total number of solutions is $\binom{16+3}{3} = \frac{19!}{3!(19-3)!}$

$= 969$ total possible solutions

$$\overbrace{\underline{x}\ \underline{x}\ \underline{x}\ \underline{x}}^{x}\ \underline{|}\ \overbrace{\underline{x}\ \underline{x}\ \underline{x}\ \underline{x}}^{y}\ \underline{|}\ \overbrace{\underline{x}\ \underline{x}\ \underline{x}\ \underline{x}}^{w}\ \underline{|}\ \overbrace{\underline{x}\ \underline{x}\ \underline{x}\ \underline{x}}^{z}$$

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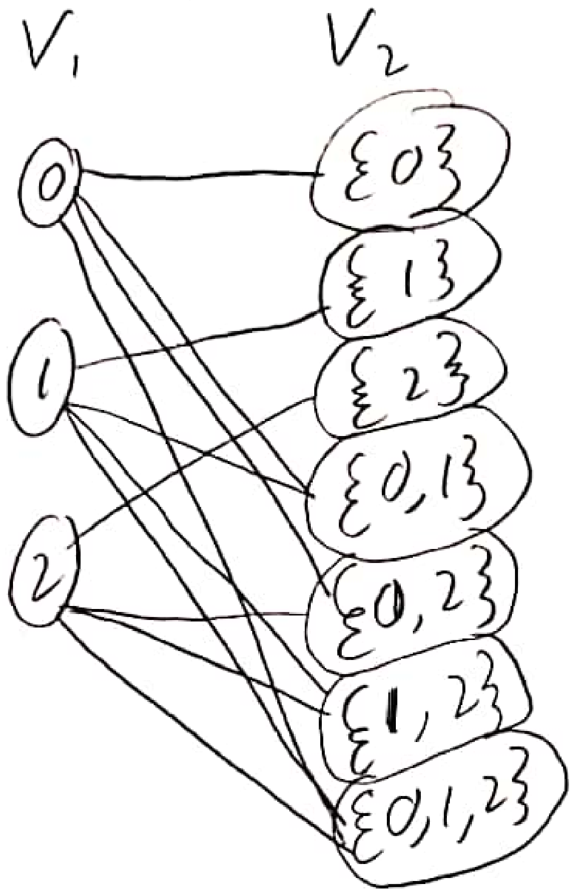
3. G



1. is G connected?

Yes. Every vertex in V_2 connects to at least one member of V_1. Every member of V_1 can be found in $\{0,1,2\}$, which is a member of $V2$. By the transitive property of path connection, there exists a path from any vertex to any vertex. $\therefore$ G is connected.

2. all vertices in V_1 connect to 4 vertices in V_2, so the degree of any $v_1 \in V_1$ is 4

3. for any $v_2 \in V_2$ there is an edge for each member of V_2. Therefore the degree of V_2 is eqlal to $|V_2|$

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4. r being a root to $x^4-c_1x^3-c_2x^2-c_3x-c_4$

$\Rightarrow r^4-c_1r^3-c_2r^2-c_3r-c_4=0$

if $a_n=r^n$

$\Rightarrow a_n=c_1a_{n-1}+c_2a_{n-2}+c_3a_{n-3}+c_4a_{n-4}$

becomes

$r^n=c_1r^{n-1}+c_2r^{n-2}+c_3r^{n-3}+c_4r^{n-4}$

$\Rightarrow r^n=r^n-(r^n-c_1r^{n-1}-c_2r^{n-2}-c_3r^{n-3}-c_4r^{n-4})$

$\Rightarrow r^n=r^n-r^{n-4}(r^4-c_1r^3-c_2r^2-c_3r-c_4)$

substitute

$\Rightarrow r^n=r^n-r^{n-4}(0)$

$\Rightarrow r^n=r^n$ checks out ✓

$\therefore$ $a_n=r^n$ is a solution to the given recurrence relation

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