MATH 61: MIDTERM 2

1

a) Paths of type $1 \to a \to 2$, where $a \in \{1', ..., 5'\}$: 5 possibilities.

b) Paths of type $1 \to a \to b \to 2'$, where $a \in \{1', 3', 4', 5'\}$ and $b \in \{2, 3, 4\}$: $4 \cdot 3 = 12$ possibilities.

c) There is no path P_4 that starts and ends in the same vertex class, so the answer is 0.

d) Paths of type $1 \to a \to b \to c \to 2$, where $a \neq c \in \{1', ..., 5'\}$ and $b \in \{3, 4\}$: $5 \cdot \cdot 2 \cdot 4 = 40$ possibilities.

2

a) The isomorphism is given by a bijection $1 \to a$, $2 \to b$, $3 \to e$, $4 \to d$, $5 \to c$, $6 \to h$, $7 \to g$, $8 \to f$.

b) The two graphs are not isomorphic. The left one contains a 3-cycle (and 5-cycle), while the right one does not.

3

The answer is $a_n = 3^n - 2^n$. All points were given regardless whether you guessed correctly and then gave proof by induction or via standard method of solving LHRR.

4

a) Take a vertex A, it has degree 1, so it is connected to some other vertex B. B also has degree 1, so it is only connected to A. Therefore no path starting at A can leave the pair $\{A, B\}$, in particular there cannot be a path between A and some other vertex C. Hence a simple graph with this degree sequence cannot be connected.

b) The sum of the degrees is odd, so there is no graph with this degree sequence.

c) The desired graph is $K_{1,5}$.

d) There is no such graph. Proof by contradiction. Assume such graph exists. Each of the four vertices of degree 5 is adjacent to all other vertices. Thus, the degree of each vertex is at least 4, a contradiction.

5

(1) T (2) F (3) T (4) F (5) F (6) F (7) T (8) T (9) T (10) T (11) T (12) F (13) F (14) F (15) T