Practice Midterm 1

UCLA: Math 61, Winter 2018

Instructor: Jens Eberhardt
Date: 02 February 2017

- This exam has 4 questions, for a total of 42 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.

Name: ______________________________

ID number: ______________________________

Discussion section (please circle):

Day/TA	HUNT, CHRISTOPHER	HAN, KYUTAE	MENEZES, DEAN
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	12	
2	10	
3	12	
4	8	
Total:	42	

Please note! The following two pages will not be graded. You must indicate your answers ***here*** *for them to be graded!*

Question 1.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) Let $a_n = a_{n-1} + 5$ and $a_0 = 5$. Then a_{100} equals

   A. $100 \times 5 + 5$
   B. $101 \times 5 + 5$
   C. 5×5^{100}
   D. 5×5^{101}

   (b) (2 points) Let n be a postive integer. Then

$$\sum_{i=0}^{n} C(n,i)5^i$$

   equals

   A. 5^{n+1}
   B. 6^{n+1}
   C. 5^n
   D. 6^n

   (c) (2 points) Let X be a non-empty finite set. Then every function $f : X \times X \to X$ is

   A. injective
   B. surjective
   C. not injective
   D. not surjective

(d) (2 points) Let $G = (V, E)$ be a graph and $v \in V$ a vertex in G. Then the degree of v, denoted by $\delta(v)$, is the number of

A. vertices adjacent to v
B. cycles containing v
C. edges incident to v
D. paths starting at v

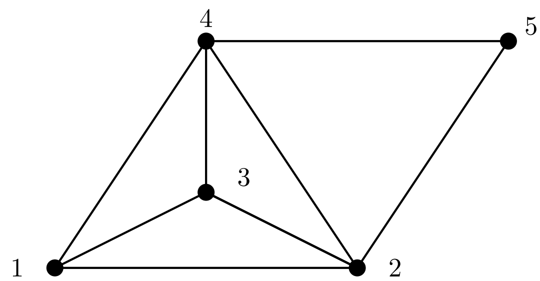
(e) (2 points) The Fibonacci numbers f_n are given by the recurrence relation $f_n = f_{n-1} + f_{n-2}$. Which of the following equalities holds?

A. $f_n = 2f_{n-2} + f_{n-3} + f_{n-4}$
B. $f_n = f_{n-2} + 2f_{n-3} + 3f_{n-4}$
C. $f_n = f_{n-2} + f_{n-3} + 2f_{n-4}$
D. $f_n = f_{n-2} + 2f_{n-3} + f_{n-4}$

(f) (2 points) Let $G = K_n$ be the complete graph on n vertices. Then G has an Euler cycle if and only if

A. n is even
B. $n = 2$
C. n **is odd**
D. $n = 3$

2. In the following questions, simply write down your answer. There is *no justification needed*. You can specify paths in simple graphs by a sequence of vertices.
Consider the following graph G.



(a) (2 points) Find a path from 1 to 5.

(b) (2 points) Find a cycle with 6 edges.

(c) (2 points) Compute

$$\sum_{i=1}^{5} \delta(i)$$

(d) (4 points) An *Euler path* is a path in G which contains every edge of G exactly once. Find an Euler path in G starting with $(1, 3, 2, 1, \dots)$.

Solution:

(a) $(1, 4, 5)$

(b) $(1, 2, 4, 5, 2, 3, 1)$

(c) Let m be the number of edges in G. Then

$$\sum_{i=1}^{5} \delta(i) = 2m.$$

In this case $m = 8$.

(d) For example: $(1, 3, 2, 1, 4, 2, 5, 4, 3)$

(e) $C(5, 3)$

3. (6 points) Consider the following recurrence relation

$$a_n = 6a_{n-1} - 9a_{n-2}$$

with initial conditions

$$a_0 = 0, a_1 = 3.$$

Solve the recurrence relation in two steps.

(a) (2 points) Determine the characteristic polynomial and its roots.

(b) (2 points) Determine the general solution.

(c) (2 points) Determine the solution fulfilling the initial conditions.

Solution:

(a) (2 points) The characteristic polynomial is

$$x^2 - 6x + 9 = (x-3)^2$$

and has the root $x = 3$ with multiplicity 2.

(b) (2 points) Since the characteristic polynomial has a multiple root the general solution has the form

$$S_n = a3^n + bn3^n$$

for real numbers a, b.

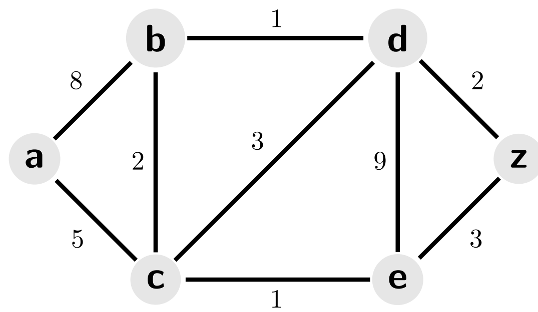
(c) (2 points) We solve

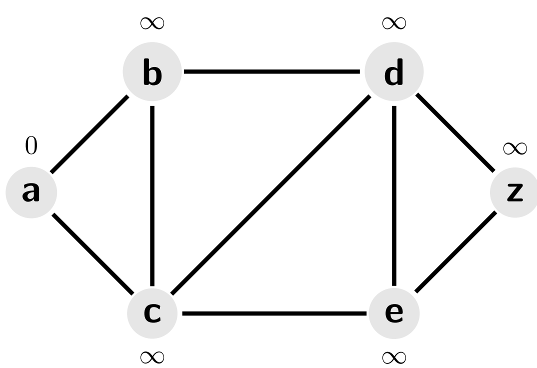
$$0 = S_0 = a$$
$$3 = S_1 = 3a + 3b$$

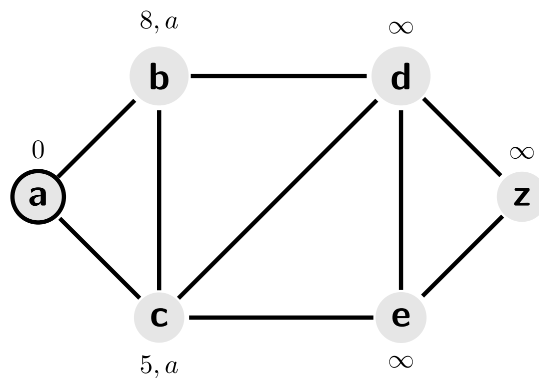
for a, b. We get $a = 0$ and $b = 3$. Hence

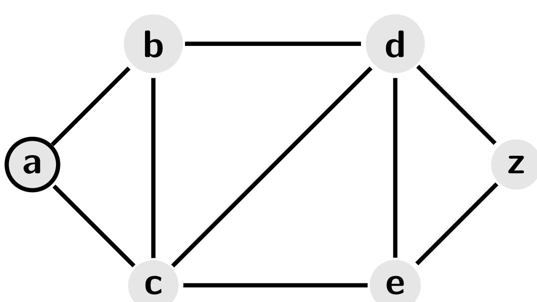
$$a_n = 3n3^n$$

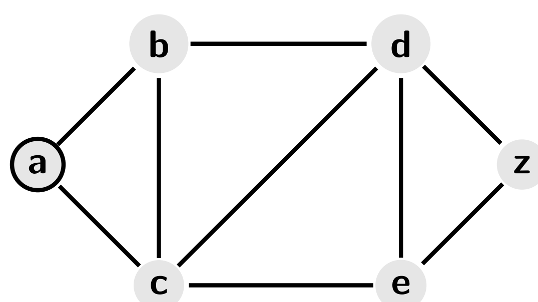
4. (8 points) Apply the next **two** iterations of Dijkstra's algorithm to find the shortest path from a to z in the following graph. In each step, annotate each vertex x with $L(x)$ and $P(x)$, as shown. Circle the vertices already visited. Use the provided blank graphs. If you make a mistake, clearly cross it out and continue using the next blank graph.

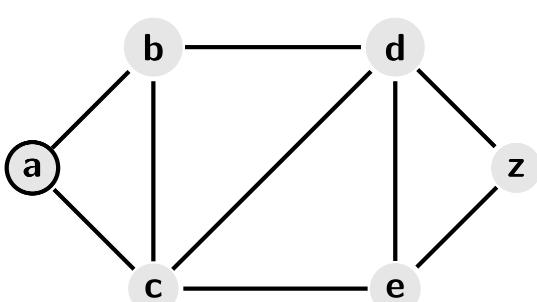


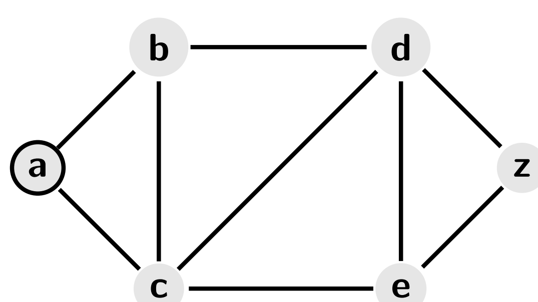




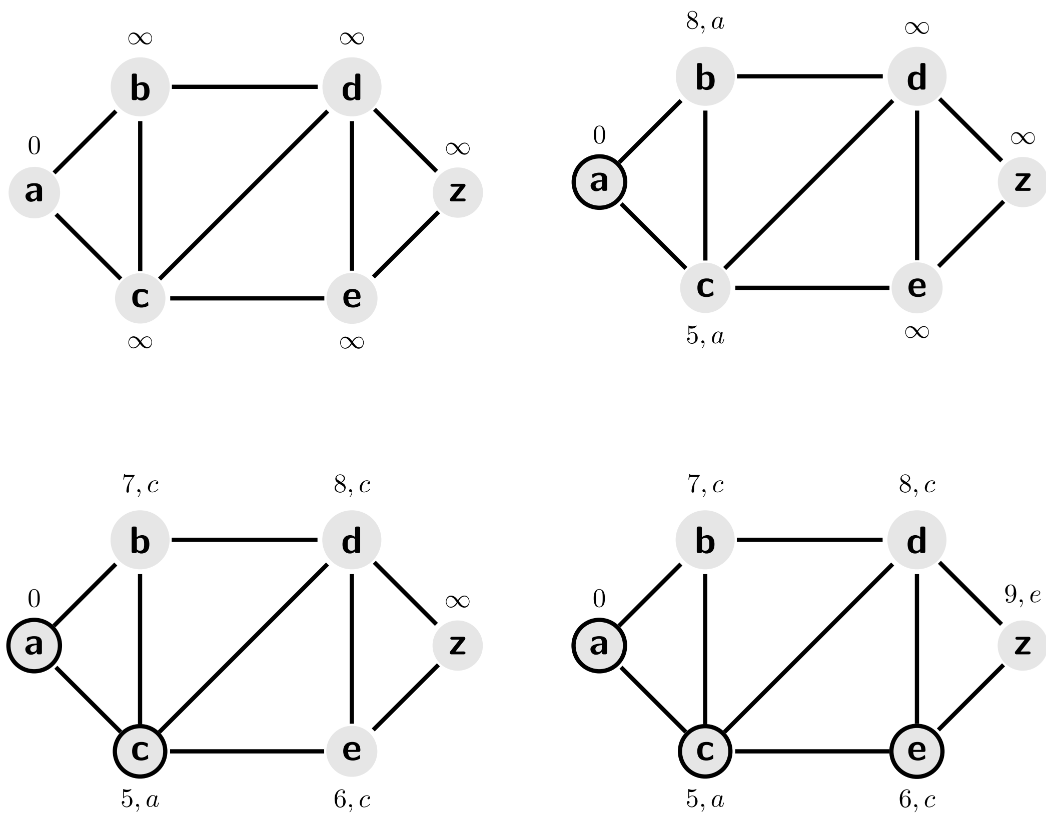








Solution:


a b c d e z
0 ∞ ∞ ∞ ∞ ∞
a b c d e z
0 8, a 5, a ∞ ∞ ∞
a b c d e z
0 7, c 5, a 8, c 6, c ∞
a b c d e z
0 7, c 5, a 8, c 6, c 9, e

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