Math 33B-2, Fall 2012, Quiz 2 (Th)

Section ____________ Name KEY

**Q1 (4 pts).** Find the exact solution to the initial value problem

$$y'' + 12y' + 36y = 0 \qquad \text{with} \qquad y(0) = 2, y'(0) = -8.$$

$$\lambda^2 + 12\lambda + 36 = 0$$

$$\lambda = -6$$

$$y(t) = c_1 e^{-6t} + c_2 t e^{-6t}$$

$$y'(t) = -6c_1 e^{-6t} + c_2(1-6t)e^{-6t}$$

$$\begin{cases} 2 = c_1 + 0 \\ -8 = -6c_1 + c_2 \end{cases}$$

$$c_1 = 2$$

$$c_2 = 4$$

$$y(t) = 2e^{-6t} + 4te^{-6t}$$

**Q2 (6 pts).** A 10-kg mass streches a spring 1 m. The system is placed in a viscous medium that provides a damping constant $\mu = 20$kg/s. The system is at spring-mass equilibrium. Then a sharp tap to the mass imparts an instantaneous downward velocity of 1.2m/s. Find the amplitude, frequency and phase of the resulting motion.

$$mg = kx_0$$

$$k = \frac{10 \cdot 10}{1} = 100 \ kg/s^2$$

$$my'' = -ky - \mu y'$$

$$y'' + 2y' + 10y = 0$$

with $y(0) = 0$

$y'(0) = 1.2$

$$\lambda^2 + 2\lambda + 10 = 0$$

$$\lambda = -1 \pm 3i$$

$$y(t) = e^{-t}(c_1 \cos 3t + c_2 \sin 3t)$$

$$y'(t) = (-c_1 \cos 3t - c_2 \sin 3t - 3c_1 \sin 3t + 3c_2 \cos 3t)e^{-t}$$



$$\begin{cases} 0 = c_1 \\ 1.2 = 3c_2 \end{cases}$$

$$\Rightarrow y(t) = e^{-t} \cdot 0.4 \sin 3t = 0.4e^{-t} \cos(3t - \tfrac{\pi}{2})$$

Amplitude $= 0.4e^{-t}$, frequency $= 3$, phase $= \frac{\pi}{2}$.