Math 33B-2, Fall 2012, Quiz 1 (Th)

Section __________ Name KEY

**Q1 (5 pts).** Find the exact solution to the initial value problem

$$\frac{dy}{dx} = \frac{3x^2}{y + x^3 y}, \qquad y(0) = -2.$$

$$y\, dy = \frac{3x^2}{1+x^3}\, dx$$

$$\frac{1}{2} y^2 = \ln|1+x^3| + C$$

$x=0,\ y=-2$

$$2 = 0 + C$$

$$C = 2$$

$$\frac{1}{2} y^2 = \ln|1+x^3| + 2$$

$$y = \pm\sqrt{2\ln|1+x^3| + 4}$$

$y(0) = -2 \Rightarrow$

$$y = -\sqrt{2\ln|1+x^3| + 4}$$

**Q2 (5 pts).** Solve the following initial value problem. Discuss the interval of existence.

$$xy' + 2y = e^x, \qquad y(2) = 0.$$

$y' + \frac{2}{x} y = \frac{1}{x} e^x$

$uy' + \frac{2}{x} uy = \frac{1}{x} u e^x$

let $(uy)' = uy' + \frac{2}{x} uy$

$u' = \frac{2}{x} u$

$u = e^{\int \frac{2}{x} dx} = x^2$

$(x^2 y)' = \frac{1}{x} \cdot x^2 \cdot e^x = x e^x$

$x^2 y = xe^x - e^x + c$

$x = 2, \ y = 0 \Rightarrow$

$0 = 2e^2 - e^2 + c$

$c = -e^2$

$y = \dfrac{xe^x - e^x - e^2}{x^2}$



interval of existence.

$(0, +\infty)$