Math 33B
Fall 2015
Midterm1
10/21/15
Time Limit: 50 Minutes

Print Name:

TA name ———— section number————

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem. **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit. Formula that you might or might not need:

Do not write in the table to the right.

Problem	Points	Score
1	20	18
2	20	19
3	20	19
4	20	18
5	20	20
Total:	100	94

Signature: ____________________ Date_______

1. (20 points) Find the exact solution of the initial value problem. Indicate the interval of existence.

$$y' = x/(1+2y), \quad y(-1) = 0.$$

$\frac{dy}{dX} = \frac{X}{1+2y}$

$(1+2y)\,dy = X\,dX$

$\int (1+2y)\,dy = \int X\,dx$

$y + y^2 + C = \frac{1}{2}X^2$

$y(-1) = 0$

$C = \frac{1}{2}(-1)^2 = \frac{1}{2}$

$y + y^2 + \frac{1}{2} = \frac{1}{2}X^2$

$y^2 + y + \frac{1}{2} - \frac{1}{2}X^2 = 0$ ✓ +10

$y = \frac{-1 \pm \sqrt{1-(2-2X^2)}}{2}$ ✓ +5

$= \frac{-1 \pm \sqrt{2X^2-1}}{2}$

interval of existence

Because $y \neq \frac{-1}{2}$

$\frac{1}{4} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2}X^2 = 0$

$X^2 = \frac{1}{2}$ $X = \pm\frac{\sqrt{2}}{2}$

So: $X \neq \pm\frac{\sqrt{2}}{2}$

$X \in \left(-\infty, -\frac{\sqrt{2}}{2}\right)$ and $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{\sqrt{2}}{2}, \infty\right)$

+3

18/20

2. (20 points) Find the solution of the initial value problem.

$$(1+t^2)y' + 4ty = (1+t^2)^{-2}, \quad y(1) = 0.$$

Integrating factor:

$u = t^2+1$

$du = 2t$

$$\int \frac{4t}{t^2+1}\,dt$$

$$= \int \frac{2\,du}{u}$$

$$= 2\ln|u| + C$$

$$= 2\ln(t^2+1) + c$$

$$e^{2\ln(t^2+1)} = (t^2+1)^2$$

$$y' + \frac{4t}{t^2+1}y = (1+t^2)^{-3}$$

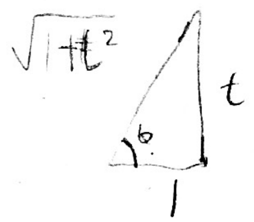
$$(t^2+1)^2 y' + 4t(t^2+1)y = \frac{1}{t^2+1}$$

$$\frac{d}{dt}\left[(t^2+1)^2 y\right] = \frac{1}{t^2+1}$$

$$\int \frac{d}{dt}\left[(t^2+1)^2 y\right] dt = \int \frac{1}{t^2+1}\,dt$$

$$(t^2+1)^2 y = \arctan t + c$$

$$y = \frac{1}{(t^2+1)^2}(\arctan(t) + c)$$



~~θ = arc~~

~~cos²θ~~

$$y(1) = \frac{1}{4}(\arctan 1 + c) = 0$$

$$\frac{\pi}{4}$$

$$= \frac{1}{4}\left(\frac{\pi}{2} + c\right) = 0$$

$$c = -\frac{\pi}{2}$$

$$y = \frac{1}{(t^2+1)^2}\left(\arctan t - \frac{\pi}{2}\right)$$

19/20

3. (20 points) Find the integrating factor to make the following equation into an exact equation. Then find the general solution. (If you remember the integrating factor, you can use it directly.)

19/20

$$(x^2y^2 - 1)ydx + (1 + x^2y^2)xdy = 0.$$

$$\frac{x^2y^2-1}{x\cancel{y}}\cancel{y}\,dx + \frac{1+x^2y^2}{\cancel{x}y}\cancel{x}\,dy = 0$$

$$\frac{\partial F}{\partial x} = \frac{x^2y^2-1}{x} \qquad \frac{\partial F}{\partial y} = \frac{1+x^2y^2}{y}$$

$$F = \int \frac{x^2y^2-1}{x}\,dx + \phi(y) = \int \left(xy^2 - \frac{1}{x}\right)dx + \phi(y)$$

$$= \frac{1}{2}x^2y^2 - \ln(x) + \phi(y) \quad ✓$$

$$\frac{\partial F}{\partial y} = x^2y + \phi'(y) = \frac{1+x^2y^2}{y}$$

$$\phi'(y) = \frac{1}{y} \qquad \phi(y) = \ln|y| + C \quad ✓$$

$$F = \frac{1}{2}x^2y^2 - \ln|x| + \ln|y| + C = 0$$

-1

4. (20 points) Suppose that x is a solution to the initial value problem

18/20

$$x' = \frac{x^3 - x}{1 + t^2x^2}, \quad x(0) = 1.$$

$$\begin{cases} x' = x - t^2 + 2t \\ x(0) = 1 \end{cases}$$

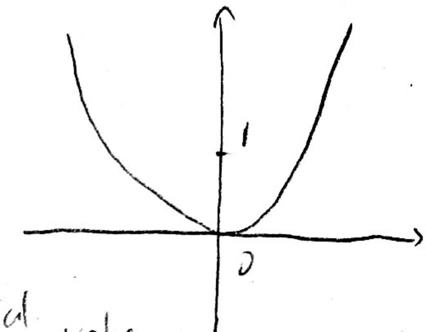
Show that $x(t) > t^2$ for all t for which x is defined.

$x'(t)$ is continous everywhere

$x(t) = t^2$ is one solution of the differential function, because

$$2t = t^2 - t^2 + 2t$$

And at $t = 0$, $x(0) = 0^2 = 0$ ✓



So at $x = 0$, the initial value of this Initial value problem is longer than the initial value of $x(t) = t^2$

The solution $x(t)$ is above the curve t^2.

According to Uniqueness thereom, there is only one solution to a unique initial value problem, in other words, two solutions of differential equation cannot cross each othe. Since $x(t) > t^2$ at t $x(t)$ is alway larger then t^2

did you check $\frac{\partial f}{\partial x}$ is continuous?

-2

5. (20 points) Find the general solution for the following differential equation.

$$4y'' + 4y' + y = 0.$$

characteristic equation:

$$4\lambda^2 + 4\lambda + 1 = 0$$

$$\lambda^2 + \lambda + \frac{1}{4} = 0$$

$$(\lambda + \tfrac{1}{2})^2 = 0$$

$$\lambda = -\frac{1}{2}$$

$$y_1 = e^{-\frac{1}{2}t}$$

$$y_2 = te^{-\frac{1}{2}t}$$

general solution

$$y = C_1 e^{-\frac{1}{2}} + C_2 t e^{-\frac{1}{2}}$$