33B Midterm 2

Nikki Kam Yee Woo

TOTAL POINTS

37 / 40

QUESTION 1

auto. Equation 11 pts

1.1 Phase Line 3 / 3

- ✓ **- 0 pts** Correct
- **- 1 pts** arrows missing/wrong
- **- 1 pts** graph wrong somewhere
- **- 1 pts** missing scale
- **- 3 pts** not the phase line
- **- 1 pts** too many zero's in the graph

1.2 Eq. Solutions 3 / 3

- ✓ **- 0 pts** Correct
- **- 1 pts** -3 no conclusion
- **- 1 pts** 2 is stable
- **- 1 pts** 5 is unstable

1.3 Graph sketch 2 / 2

- ✓ **- 0 pts** Correct
- **- 2 pts** no solution/wrong solution
- **- 1 pts** graph between -3 and 2 wrong/missing
- **- 1 pts** graph between 2 and 5 wrong/missing

1.4 particular solution 3 / 3

- ✓ **+ 3 pts** Correct
- **+ 1 pts** No
- **+ 1 pts** Uniqueness theorem can be applied
- **+ 0 pts** wrong/no answer
- **+ 1 pts** cannot cross the equilibrium solution y(t) = 2

QUESTION 2

Existence and Uniqueness 8 pts

2.1 Apply? Rectangle? 4 / 5

- ✓ **+ 2 pts** continuous
- ✓ **+ 2 pts** derivative continuous
- ✓ **+ 1 pts** rectangle
- **+ 0 pts** no points
- **- 1** Point adjustment

x can also be less than -2

2.2 x_0(2)=5? 3 / 3

- ✓ **+ 1 pts** Correct
- ✓ **+ 2 pts** justification
- **+ 0 pts** no points

QUESTION 3

3 Particular Solution 6 / 6

- ✓ **- 0 pts** Correct
- **- 1 pts** Mixed up a minus sign
- **- 3 pts** Didn't try the right guess (ae^3t)
- **- 6 pts** Didn't attempt method of undetermined coefficients.
- **- 1 pts** Incorrect division
- **- 1 pts** Incorrect multiplication
- **- 1 pts** Put constant in solution
- **- 3 pts** Forgot to include an undetermined coefficients in MOC.

QUESTION 4

2. order equation constant coefficients 5 pts

4.1 verify solutions 3 / 3

- ✓ **- 0 pts** Correct
- **- 2 pts** Didn't explicitly check boundary conditions
- **- 1 pts** Only checked one boundary condition
- **- 1 pts** Didn't correctly check that they satisfy the ODE.

4.2 existence and uniqueness? 0 / 2

- **- 0 pts** Correct
- **- 2 pts** Didn't understand that solution was non-unique.
- ✓ **- 2 pts** Didn't state that the boundary conditions being defined at different times renders the existence and uniqueness theorem irrelevant.

- **1 pts** Not clear if you actually meant that the "initial" conditions are defined at different times.

QUESTION 5

2. order equation 7 pts

5.1 verify solutions **4 / 4**

✓ **- 0 pts** **Correct**

- 2 pts incorrect calculation

- 4 pts incorrect calculation

5.2 fundamental set **3 / 3**

✓ **- 0 pts** **Correct**

- 1 pts conclusion is incorrect,

- 1 pts some work, calculation incorrect,

- 3 pts conclusion incorrect, wrong calculation

- 2 pts some work

QUESTION 6

6 planar system **3 / 3**

✓ **- 0 pts** **Correct**

- 2 pts incorrect, but some work

- 1 pts minor mistake

- 3 pts no work

195

MIDTERM 2

11/16/2018

Math33B
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section: 2A

Problem	Points	Score
1	11	
2	8	
3	6	
4	5	
5	7	
6	3	
Total	40	

Instructions

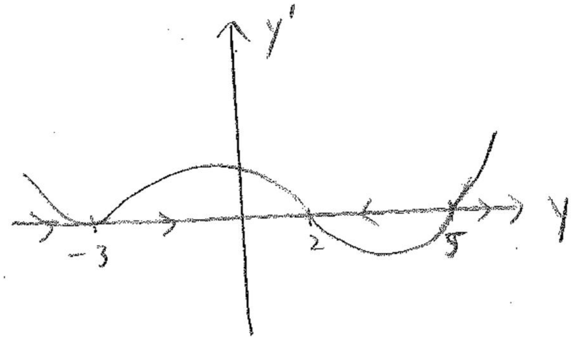
(1) This exam has 6 problems. Make sure you have all pages.
(2) Enter your name, SID number, and discussion section on the top of this page.
(3) **Use a PEN** to record your final answers.
(4) If you need **more space**, use the extra page at the end of the exam.
(5) NO Calculators, computers, books or notes of any kind are allowed.
(6) Show your work. Unsupported answers will not receive full credit.
(7) Good Luck!

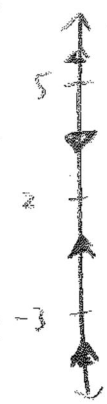
Exercise 1. (11pt)
Consider the autonomous first-order differential equation.

$$y' = (y+3)^2(y-2)(y-5)$$

(1) Draw the phase line. (3pt)

zeros: $-3, 2, 5$





(2) What are the equilibrium solutions? Which are stable, and which are unstable? (3pt)

equilibrium solns:

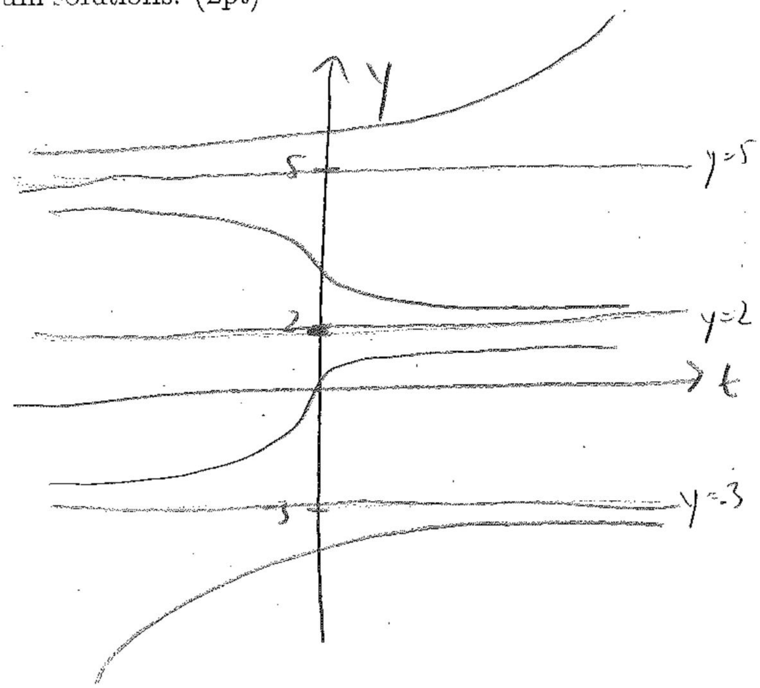
$y = -3, 2, 5$

stable: 2

unstable: 5

undetermined: -3

(3) Sketch the graph of at least one solutions between each pair of adjacent equilibrium solutions. (2pt)



(4) Let $y_p(t)$ be a particular solution to the equation which satisfies $y_p(0) = 0$. Is it possible that $y_p(2) = 2$? Justify your answer. (3pt)

$$y' = (y+3)^2(y-2)(y-5) \quad (*) \quad = f(y)$$
$$f'(y) = 2(y+3)$$

NO, since we know (*) is defined & continuous on all reals and $f'(y)$ is continuous on all reals, the existence & uniqueness th. are satisfied. Because of this, we know that for the part. soln $y_p(t)$ which satisfies $y_p(0)=0$, this solution must stay in the bounds of the equilibrium solns due to uniqueness. Thus, it is not possible for $y_p(2)=2$ to be satisfied because y_p must stay in between the bounds $y=2$ and $y=-3$.

Exercise 2. (8pt) Consider the differential equation

$$\frac{dx}{dt} = \frac{\sqrt{x^2-4}}{t^2}$$

(1) Can you apply the existence and uniqueness theorem to the initial value problem $x_0(1) = 6$? Justify your answer and give the biggest rectangle in which you can apply it containing the given point (if it exists). (5 pt)

$f(x,t) = \frac{dx}{dt} = \frac{\sqrt{x^2-4}}{t^2}$ this function is defined & continuous when $t \neq 0$ and $x \geq 2$

$\frac{\partial f}{\partial x} = \frac{1}{t^2}\left(\frac{1}{2}\right)(x^2-4)^{-\frac{1}{2}}(2x) = \frac{x}{t^2\sqrt{x^2-4}}$ this function is continuous for $x \neq 2, -2$, & $t \neq 0$

for the initial value problem $x_0(1) = 6$

we see that, for $x=6$ & $t=1$, $f(x,t)$ and $\frac{\partial f}{\partial x}$ are continuous and $f(x,t)$ is defined. Thus, we can apply the existence & uniqueness theorem to this tuple. However, we see that $\frac{\partial f}{\partial x}$ is not continuous at $x = 2, -2$ and $t = 0$ and $f(x,t)$ is not defined or continuous at $t = 0$ and $x \geq 2$

thus, the largest rectangle we can apply existence & uniqueness to including the tuple $(1, 6)$ is

$$\boxed{\begin{array}{l} t \in (0,\infty) \\ x \in (2,\infty) \end{array}}$$

(2) Can $x_0(2) = 5$ ($x_0(t)$ is the solution to the initial value problem in part 1))?(1pt) Justify your answer. (2pt)

$x_0(2) = 5$

No, in order to go from (1, 6) to (2, 5) the slope would have to be negative and thus for some values between $x = 5$ and $x = 6$, $\frac{dx}{dt}$ would have to have a negative value. Since the equation for $\frac{dx}{dt}$ is $\frac{\sqrt{x^2-4}}{t^2}$, there is no way of plugging in an x and a t such that $\frac{dx}{dt}$ will be negative. And, since we defined earlier that the existence & uniqueness theorem applies for the region $x \in (2, \infty)$ & $t \in (0, \infty)$ we know that $x_0(1) = 6$ is a unique soln for that region. Given that information, there is no way that the solution satisfying $x_0(1) = 6$ can also satisfy $X_0(2) = 5$.

Exercise 3. (6pt) Find a **particular** solution to the following differential equation

$$3y'' + 2y' - y = -4e^{3t}.$$

$$3y'' + 2y' - y = -4e^{3t}$$

part soln takes the form $y_p = ae^{3t}$.

$y_p = ae^{3t}$ $y_p' = 3ae^{3t}$ $y_p'' = 9ae^{3t}$

$$27ae^{3t} + 6ae^{3t} - ae^{3t} = -4e^{3t}$$

$$32ae^{3t} = -4e^{3t}$$

$$a = -\frac{1}{8}$$

$$\boxed{y_p = -\frac{1}{8}e^{3t}}$$

$\frac{\pm\sqrt{-4}}{2}$ = $\pm i$

Exercise 4. (5pt) Consider the following problem:

$$y'' + y = 0 \qquad y(0) = 0 \quad y'(\pi/2) = 0$$

(1) Show that $y(t) = C \cdot \sin(t)$ is a solution for any constant C. (3pt)

$y(t) = C\sin(t) \quad y' = C\cos(t) \quad y'' = -C\sin(t)$

$-C\sin(t) + C\sin(t) = 0$

$0 = 0$ ✓

$y(0) = 0 = C\sin(0) = 0$ ✓

$y'(\pi/2) = 0 = C\cos(\pi/2) = 0$ ✓

(2) Why does this not violate the 2. order existence and uniqueness theorem? (2pt)

This does not violate the 2nd order existence and uniqueness theorem because, in the given second order ODE, $\frac{dy}{dt}$ is not defined, and thus not continuous, and thus the uniqueness theorem is not satisfied. Thus, the solutions do not have to be unique and $y(t) = C\sin(t)$ is free to have any constant.

$2(x^2+3x+2)$

$2(x+2)(x+1)$

Exercise 5. (8pt) Consider the differential equation

$$y'' + \frac{1+x}{x}y' - \frac{1}{x}y = 0$$

(1) Check that 1) $1+x$ and 2) $\frac{2x^2+6x+4}{x+2}$ are solutions to the above equation. (4pt)

1) $y = 1+x \quad y' = 1 \quad y'' = 0$

$$0 + \frac{1+x}{x}(1) - \frac{1}{x}(1+x) = 0$$

$$\frac{1+x}{x} - \frac{1+x}{x} = 0 \checkmark$$

2) $y = \frac{2x^2+6x+4}{x+2} = \frac{2(x+2)(x+1)}{x+2} = 2(x+1)$

$y = 2(x+1) \quad y' = 2 \quad y'' = 0$

$$0 + \frac{1+x}{x}(2) - \frac{1}{x}(2(x+1)) = 0$$

$$\frac{2(1+x)}{x} - \frac{2(x+1)}{x} = 0 \checkmark$$

(2) Do they form a fundamental set of solutions?(1pt) Justify your answer. (2pt)

No, $\frac{2x^2+6x+4}{x+2}$ simplifies to $2(x+1)$

which is a constant multiple of $(x+1)$

thus, they are linearly dependent and do not form a fundamental set of solns.

to further prove that the two are linearly dependent, we can turn to the wronskian

$$W = \det\begin{pmatrix} x+1 & 2(x+1) \\ 1 & 2 \end{pmatrix} = 2(x+1) - 2(x+1) = 0$$

since the wronskian $=0$, we know the two solns are linearly dependent and thus do not form a fundamental set of solutions.

Exercise 6. (3pt)
Consider the second order equation

$$y'' - 2e^t y' - \tan(t)y = \sqrt{t^2+1}.$$

Write this equations as a planar system of first-order equations.

$$y'' - 2e^t y' - \tan(t)\, y = \sqrt{t^2+1}$$

$$y'' = v'$$

$$y' = v$$

$$v' = 2e^t v + \tan(t)\, y + \sqrt{t^2+1}$$

Extra page

Extra page