33B midterm 1

Nikki Kam Yee Woo

TOTAL POINTS

29.5 / 40

QUESTION 1

integration factor 8 pts

1.1 integration factor 4 / 4

✓ **- 0 pts Correct**

- 1 pts minor mistake

- 4 pts no work

- 3 pts subtle work, try to find h(x) but equation incorrect

- 2 pts get h(x),but not u(x)

- 2 pts get u(x) but without details; know how to get u(x) but calculate incorrectly

1.2 solve 4 / 4

✓ **- 0 pts Correct**

- 1 pts solution should be in form of F(x,y) = c

- 4 pts no work

- 3 pts know need to do partial integration, but incorrect.

- 2 pts correct form F = \phi + xxxx, but \phi incorrect ; or the other way around.

- 1 pts minor mistake

QUESTION 2

separable eon 12 pts

2.1 explicit solution 1 / 5

✓ **+ 1 pts Separating the Equation**

+ 1 pts Partial Fractions

+ 1 pts Computing Integral

+ 1 pts Log Rule Application

+ 1 pts Computing Solution

+ 2 pts Bernoulli Transformation

+ 1 pts Integrating Factor

+ 2 pts Rest of Bernoulli Solution

+ 0 pts No points

Use partial fractions to compute the integral

2.2 y(1) = 2 0 / 2

+ 2 pts Correct Answer

+ 1.5 pts Correct Answer, Wrong Solution

+ 1 pts Knowing the Process

✓ **+ 0 pts No points**

2.3 interval of existence 0 / 3

+ 1 pts Knowing 0 is not included

+ 1 pts Correct for their function

+ 1 pts Correct

+ 1 pts Knowing 2 is not included.

✓ **+ 0 pts No points**

2.4 y(1) = 0 2 / 2

✓ **+ 2 pts Correct Answer**

+ 1 pts Correct Answer, but on accident

+ 0 pts No points

QUESTION 3

3 mixing problem 6 / 7

- 1 pts Identifying x'= rate in- rate out, rate in = 4

- 2 pts Identify rate out = x/(50+t)

- 1 pts Find an integrating factor or homogeneous solution

- 2 pts Find the general solution

✓ **- 1 pts Incorporate the initial condition.**

- 0 pts Correct

- 1 pts Accidentally made equation Homogeneous/ too simple.

- 1 pts Forgot a factor of 2 in rate out.

QUESTION 4

exact eqn 7 pts

4.1 not exact 3 / 3

✓ **- 0 pts Correct**

- 3 pts No answer

- 2 pts wrong derivatives

- **1 pts** wrong Q derivative
- **3 pts** wrong approach
- **1 pts** why?
- **1 pts** wrong P derivative

4.2 integration factor **4 / 4**

✓ **- 0 pts Correct**

- **1 pts** sign mistake
- **3 pts** only formula
- **1 pts** a=? b=?
- **4 pts** wrong/no work
- **2 pts** right start

QUESTION 5

SA 6 pts

5.1 dir field **4 / 4**

- **2 pts** No 2. solution
- **2 pts** No 1. solution
- **1 pts** mistake 1. solution
- **1 pts** mistake 2. solution
- **4 pts** doesn't go through the right points
- **2 pts** doesn't go through the right point 1. solution

✓ **+ 4 pts correct**

5.2 Y/N **1.5 / 2**

✓ **- 0.5 pts 1 incorrect**

- **1 pts** 2 incorrect
- **1.5 pts** 3 incorrect
- **2 pts** all incorrect

+ **2 pts** correct

MIDTERM 1

10/24/2018

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section: 2A

Math33B
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Problem	Points	Score
1	8	
2	12	
3	7	
4	7	
SA	6	
Total	40	

$$(2y^2+4x^2)dx + (2xy)dy = 0$$

Exercise 1. (8pt)
Consider the differential equations

$$2y^2 + 4x^2 + 2xy\frac{dy}{dx} = 0$$

(1) Find the integrating factor for the above equations.(4pt)
(Hint: it only depends on x)

$\mu(x)$

$\partial_y(\mu P) = \partial_x(\mu Q) \Rightarrow (\partial_y \mu)P + \mu\partial_y(P) = (\partial_x \mu)Q + \mu\partial_x(Q)$

$-\mu\partial_x Q + \mu\partial_y(P) = \partial_x Q \qquad \partial_x \mu = \frac{\mu}{Q}(\partial_y P - \partial_x Q)$ → $4y - 2y$

$\frac{\partial P}{\partial y} = 4y \qquad \frac{\partial Q}{\partial x} = 2y \qquad \partial_x \mu = \mu(2y)\left(\frac{1}{2xy}\right) \Rightarrow \frac{\mu'}{\mu} = \frac{1}{x}$

$\ln\mu = \ln x \qquad \mu = e^{\int \frac{1}{x}dx} = e^{\ln|x|} = x$

$$\boxed{\mu(x) = x}$$

(2) Solve the equation.(4pt)

$(*)\ (2y^2+4x^2)dx + 2x^2y\,dy = 0$

$(2xy^2 + 4x^3)dx + 2x^2y\,dy = 0$

exact? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \qquad 4xy = 4xy$ ✓

$F(x,y) = \int 2xy^2 + 4x^3\,dx + \phi(y)$

$= x^2y^2 + x^4 + \phi(y)$

$\phi'(y) = 2x^2y - \frac{\partial}{\partial y}(x^2y^2 + x^4)$

$= 2x^2y - 2x^2y = 0$

$$\boxed{F(x,y) = x^2y^2 + x^4}$$

Exercise 2. (12pt) Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2 - y}{x}$$

(1) Find the explicit general solution. (5pt)

$$\int \frac{dy}{y^2 - y} = \int \frac{dx}{x}$$

$$\frac{\ln(y-1)}{y} - \int \frac{\ln(y-1)}{y^2}$$

$$\frac{dy}{dx} = \frac{y^2 - y}{x} \qquad a(x) = \frac{1}{x}$$

$$\frac{\ln(y-1)}{y} - \int \frac{\ln(y-1)}{y^2} dy = \ln(x) + C$$

$$-\ln x$$

(2) Find the solution to this equation that satisfies the initial condition $y(1) = 2$. (2pt)

$$y(1) = 2$$

$$\ln(0)$$

(3) What is the interval of existence of the solution you found in (b). (3pt)

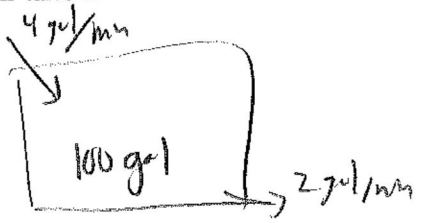
$(0, \infty)$

(4) Find the solution to this equation that satisfies the initial condition $y(1) = 0$. (2pt)

Since $y(1) = 0$ is not possible in the eqn

$y(t) = 0$ is soln

Exercise 3. (7pt) Suppose there is a tank filled with 100 gallons of water. Pure acid flows into the tank at a rate of 4 gal/min and the well mixed solution leaves the tank at the of 2 gal/min rate. Let $x(t)$ be the volume in gallons of acid in the tank at time t. Find $x(t)$ for any given time t.



$x'(t) = \text{rate in} - \text{rate out}$

$$4 - \frac{2x}{100+(2t)} = x'(t)$$

$$x'(t) = -\frac{2x}{100+(2t)} + 4$$

$$a = -\frac{2}{2(50+t)} = -\frac{1}{50+t}$$

$$f = 4$$

$$x_h = e^{\int -\frac{1}{50+t}\,dt} \qquad \text{let } u = 50+t,\ du = dt$$

$$= e^{-\int \frac{1}{u}\,du} = e^{-\ln|u|} = e^{\ln(50+t)^{-1}} = \frac{1}{50+t}$$

$$v' = \frac{f}{x_h} = \frac{4}{\frac{1}{50+t}} = 4(50+t) = \int 200 + 4t\,dt$$

$$v = 200t + 2t^2 + C$$

$$x(t) = v y_h = (200t + 2t^2 + C)\left(\frac{1}{50+t}\right)$$

$$\boxed{x(t) = \frac{2t(100+t)+C}{50+t}}$$

Exercise 4. (7pt) Consider $\underset{P}{4yxdx} + \underset{Q}{5x^2dy}$

(1) Show that the above equation is not exact. (3pt)

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$4x \neq 10x$$

not exact because $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$

(2) Find a and b such that $x^a y^b$ is an integration factor of the above equation. (4pt)

$$x^a y^b (4yx)\,dx + x^a y^b (5x^2)\,dy$$

$$= 4x^{a+1} y^{b+1}\,dx + 5x^{a+2} y^b\,dy$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$4(b+1)x^{a+1}y^b = 5(a+2)x^{a+1}y^b$$

$$4(b+1) = 5(a+2)$$

$$4b+4 = 5a+10$$

$$4b - 5a = 6$$

$4b - 5a = 6$

$-5a = 6 - 4b$

$a = \frac{6-4b}{-5}$

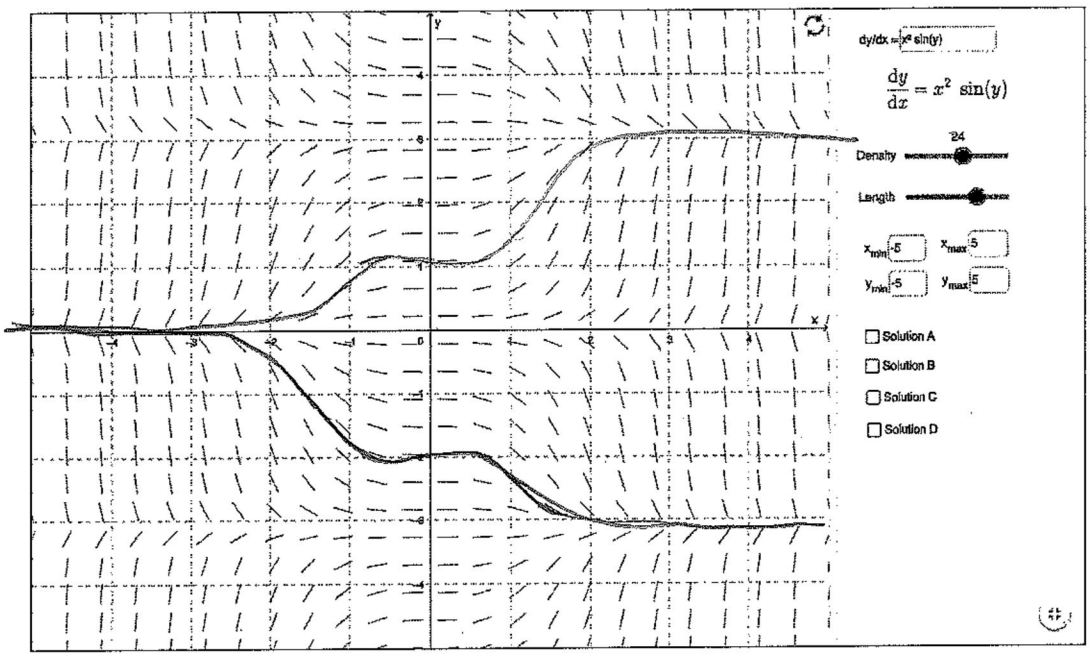
$4b = 6 + 5a$

$b = \frac{6+5a}{4}$

$a = \frac{4b-6}{5}$ $\qquad$ $b = \frac{6+5a}{4}$

ex: $a = 2$ $b = 4$

Field M1 F18.png



for $x=0$
$\frac{dy}{dx}=0$
for $y=0$
$\frac{dy}{dx}=0$
for $y=\pi$

1. Short answer problems

(no explanation needed)

(1) (4pt) Consider the above direction field and draw the solution through (0,1) and the solution through (0,-2).

(2) (2pt) Which of the following are homogeneous differential equations?

homogeneous

$P(tx,ty) = t^n P(x,y)$

$Q \cdots = Q \cdots$

Y / (N) $\sin(\frac{x}{y})dy + 2dx = 0$

Y / (N) $(xy+x^2)dy + (y^2x - x^2y)dx$

dy 2 dy 3

Y / (N) $\sin(xy)dy - \cos(xy)dx$

(Y) N $\sqrt{x^2y^2 - 4xy^3}dy + x^2dx$

dy 2