Math 33A - Lectures 3 and 4 Fall 2018

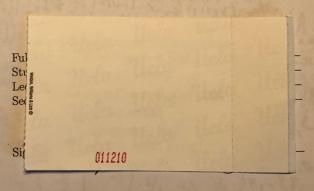
Midterm 1

Instructions: You have 60 minutes to complete this exam. There are five questions, worth a total of 50 points. This test is closed book and closed notes. No calculator is allowed.

For full credit show all of your work legibly. Unless instructed otherwise, you need to justify your answers. Please write your solutions in the space below the questions; INDICATE if you go over the page and/or use the scrap pages at the end of this booklet.

Please take a moment to ensure that your booklet consists of ten pages, the last three being reserved for additional work.

Do not forget to write your full name, section and UID in the space below. For identification purposes, please sign below.



Question	Points	Score
1	8	8
2	10	5
3	11	8
4	10	10
5	11	1
Total:	50	38

din (Im) + les (A) = M Problem 1. For each of the following sentences, give an example of a matrix A with the following properties. Or explain the following sentences, give an example of a matrix A with the following properties, or explain why it is impossible. (a) [4pts.] A is a 3×6 matrix with rank and nullity both equal to 3. PREF shows pichs in cheziles so roule=3 [=1]

X1 + X4+X5+X1=0

X2 + X4+Zx5+3x6=0

1 =0 \[\times 3+3=6

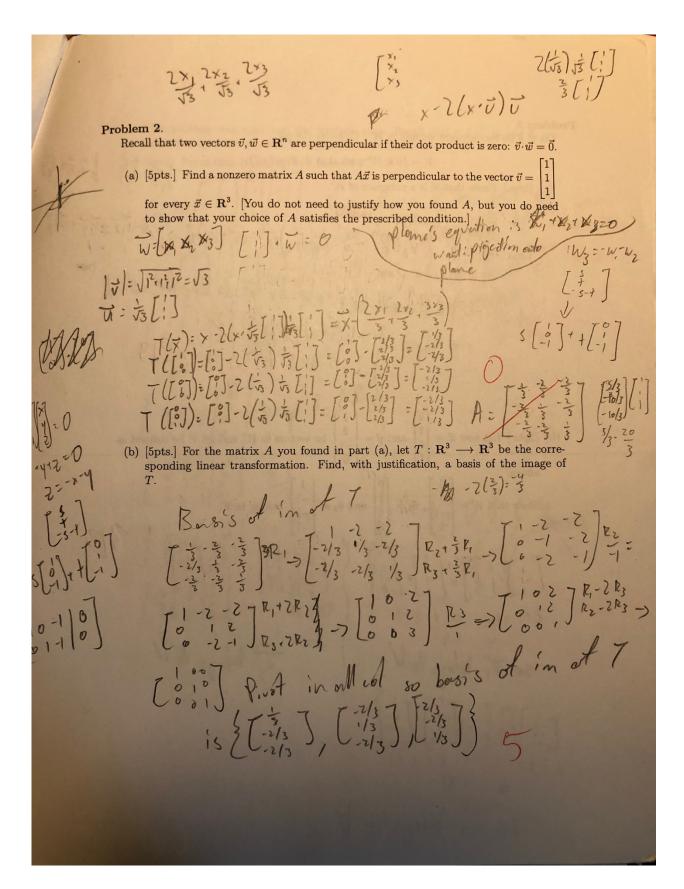
Not possible by soult-nullity theorem.

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Rank = 3, & nullity = 3 so soult + nullity = 6. However,

There are only 3 cohoms sop & 6 t 3 so it is

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Problem 3.

Let \vec{x}, \vec{y} be two nonzero vectors in \mathbb{R}^n . Consider the set

$$V = \{ \vec{v} \in \mathbf{R}^n \text{ such that } \vec{v} \cdot \vec{x} = \vec{v} \cdot \vec{y} \}.$$

(a) [3pts.] Prove that V is a subspace of \mathbb{R}^n .

have $\vec{v}_{i} \in \mathbb{R}^{n} .$ Live $\vec{v}_{i} = \vec{v}_{i} \cdot \vec$

(b) [3pts.] Let now $\vec{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ be vectors in \mathbb{R}^4 and let V be defined as

Show that $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{v}_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$ belong to V.

[:],[:]=[:]=[:]=[:] 2+1+1m = 1+2+1

so v, bilogs to V

Vy belogs to V

so v3 belons to V

This problem continues from the previous page. Recall that $\vec{x}, \vec{y}, \vec{v}_1, \vec{v}_2$ and \vec{v}_3 are defined in question (b), and $V = \{ \vec{v} \in \mathbf{R}^4 \text{ such that } \vec{v} \cdot \vec{x} = \vec{v} \cdot \vec{y} \}$.

(c) [5pts.] Prove that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis of V.

(c) [5pts.] Prove that $\{v_1, v_2, v_3\}$ is a basis of V .
V, V2 V3 = [-1][6][-1] are all in V by part brown viced to so V, V2 V3 sporm V weed to that [-1] +y[1] +z[-1] = 0 only when x=y=z=0 that so linearly independent [-1] 12 - P,
x [i] +y[i] +z[i] = 0 only when x=y=z=0 so linearly independent
[:10-10] R3-R1 each are in 119
[0 - 2 0 0 Swap R2 LR3 215
[0 10 0] R3
[80-2/0] P1-P3 [80-2/0] P1-P3 [80-2/0] P1-P3 [80-2/0] P2-P3 [80-2/0] P2-P3
[0 0 0 0 0 0 X = 0 Y = 0 X = 0

Problem 4.

(a) [5pts.] Using row-reduction find, if it exists, the inverse of the matrix

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$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

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Inverse does not exist ble PREF is not and I,

while reduced 2 \$3 [1] Let P be the plane in \mathbb{R}^3 given by the equation x - y + z = 0.

(a) [5pts.] Let $T: \mathbf{R}^3 \longrightarrow \mathbf{R}^3$ be the reflection across the plane P. Find the matrix of T with respect to the standard basis of \mathbb{R}^3 .

Problem 5.

$$T = x - 2(x \cdot l_{33}) \left[\frac{1}{2} \frac{1$$

(b) [6pts.] Find a basis $\mathfrak{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ of \mathbb{R}^3 such that the \mathfrak{B} -matrix of T is diagonal.

Write down the
$$\mathfrak{B}$$
-matrix of T .

 $V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $V_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $V_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $V_5 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $V_7 = \begin{bmatrix} 1 \\$