**MATH 33A - SECTION 2**
**PRACTICE MIDTERM #2**

**Problem 1.** Consider the linear transformation

$$T : \mathbb{R}^4 \longrightarrow \mathbb{R}^2$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \longmapsto \begin{bmatrix} 2x_1 - x_2 + 2x_3 + 4x_4 \\ 2x_1 + 3x_2 + x_3 \end{bmatrix}.$$

**(a)** Find a basis of $\ker(T)$.

**(b)** Find a basis of $\mathrm{im}(T)$.

**Problem 2**. Find $\mathrm{proj}_V(\vec{x})$, where $\vec{x} = \begin{bmatrix} 49 & 49 & 49 \end{bmatrix}^\top$ and $V$ is the kernel of the linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}$ given by $T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 6x + 2y - 3z$.

**Problem 3**. Let $A$ be a square $n \times n$ matrix.

**(a)** Show that $(A^\top)^\top = A$.

**(b)** Show that the matrix $A + A^\top$ is symmetric.

**(c)** Show that the matrix $A - A^\top$ is skew-symmetric.

**(d)** Write $A$ as the sum of a symmetrix matrix and a skew-symmetric matrix.

**Problem 4**. Compute $\det \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 2 & 2 & 4 \\ 0 & 0 & 1 & 3 & 3 & 5 \\ 0 & 0 & 0 & 1 & 4 & 6 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$.