MATH 33A - SECTION 2
PRACTICE MIDTERM #1

Problem 1. Solve the following system of linear equations.

$$\left.\begin{array}{rcr} x_1 + x_2 - x_3 + 3x_4 & = & -2 \\ x_2 - 3x_3 - x_4 & = & 6 \\ 3x_1 + 4x_2 - 6x_3 + 8x_4 & = & 0 \\ x_2 - 3x_3 - 4x_4 & = & 12 \end{array}\right\}$$

Problem 2. Find a polynomial of the form

$$f(t) = a + bt + ct^2 + dt^3$$

whose graph goes through the points $(0, 1)$, $(1, 0)$, $(-1, 0)$, and $(2, -15)$.

Problem 3. Recall that the matrix A_L associated with the reflection ref_L about a line L in $\mathbb{R}^2$ is given by

$$A_L = \begin{bmatrix} 2u_1^2 - 1 & 2u_1u_2 \\ 2u_1u_2 & 1 - 2u_1^2 \end{bmatrix},$$

where $\vec{u} = (u_1, u_2)$ is a unit vector parallel to L.

Consider the following lines in $\mathbb{R}^2$:

$$L_1 : y = 0 \qquad L_2 : y = x.$$

(a) Compute the matrix associated with $R_{L_1} \circ R_{L_2}$.

(b) Using that $R_L^2 = \mathrm{id}_{\mathbb{R}^2}$ for any line L, show that $R_{L_1} \circ R_{L_2}$ is invertible and find its inverse.

Problem 4. Find the inverse of the linear transformation $T : \mathbb{R}^5 \longrightarrow \mathbb{R}^5$ given by

$$T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 \\ x_2 + 2x_3 + 3x_4 + 4x_5 \\ x_3 + 2x_4 + 3x_5 \\ x_4 + 2x_5 \\ x_5 \end{bmatrix}$$