Math 32A/4
Fall 2012
Final Exam
December 12, 2012
90 points

Name (Print): ____________________

ID #: ____________________

TA: ____________________

This exam consists of 12 pages (including this cover page) and 9 problems. Check to see if any pages are missing.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
Total:	90	

- **Rules.** No calculators, computers, notes, books, or other aids are allowed.

- **Style.** To receive full credit, the reasoning leading to a solution must be clear and complete. A correct answer given without a complete and correct argument will be worth little or no credit.

  Organize your work. Messy and scattered work without a clear order will receive very little credit.

  In the statements of the problems, vectors are set in boldface ($\mathbf{v}$) and scalars are in plain type (v). In your solutions, please write vectors with arrows ($\vec{v}$) and scalars without arrows (v).

  There is no need to convert radicals or trigonometric functions to decimal form. For example, $\sqrt{2}$ and $\cos(50°)$ are acceptable answers. Nevertheless, simplify your answers when possible. For example, $4^{\frac{3}{2}} = 8$ and $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

- **Extra Paper.** There is a sheet of paper for scratch work at the end of the exam. If you need more space to write solutions, get more paper from the proctor. If you decide to use extra paper, please write that you have done so on the front of the exam and please staple the extra sheets to the rest of the exam.

1. (10 points) Find all critical points of the function $f(x, y) = 3x + 12y - x^3 - y^3$. Use the Second Derivative Test to determine whether each critical point is a local maximum, a local minimum, or a saddle point (or state that the the test is inconclusive).

2. (10 points) Find the absolute maximum and absolute minimum values of

$$f(x, y) = x^2 + y^2 - x$$

on the set

$$\mathcal{D} = \{(x, y) : x^2 + y^2 \leq 1\}.$$

3. Let $f(x,y)$ be a differentiable function of two variables. Suppose that $\nabla f(1,1) = \langle 3,4 \rangle$ and that $f(1,1) = 1$.

(a) (5 points) Compute

$$\lim_{s \to 0} \frac{f(2s-1, \cos(s)) - 1}{s}.$$

(b) (5 points) Let $g(x,y) = x^2 y f(x,y)$. Find a vector parametrization for the tangent line to the level curve $g(x,y) = 1$ at the point $(1,1)$.

4. (a) (3 points) Let $\mathcal{S}$ be the surface

$$z = xy^5 + \frac{1}{2}x^2y.$$

Find the tangent plane to $\mathcal{S}$ at the point $(2, 1, 4)$.

(b) (7 points) Let $\mathcal{T}$ be the surface

$$2x^2 + 2y^2 + 6y + 2xy + z^2 = 10.$$

Find all points on $\mathcal{T}$ where the tangent plane is horizontal.

5. (a) (6 points) Find the maximum value of $f(x, y) = \sqrt{xy}$ subject to the constraint $x + y = 1$, and assuming that x and y are positive.

(b) (4 points) Use the result of part (a) to show that for all positive x and y,

$$\sqrt{xy} \leq \frac{x+y}{2}.$$

6. (10 points) Suppose that $u(x, y)$ is a function of two variables which satisfies $u_{xx} = u_{yy}$. Let $x = s + t$ and $y = s - t$. Use the chain rule to show that $u_{st} = 0$. (*Note:* You may assume that all first and second order partial derivatives of u exist and are continuous.)

7. (a) (5 points) Let

$$f(x,y) = \frac{xy^4}{x^2 + y^{10}}.$$

Either compute $\lim_{(x,y)\to(0,0)} f(x,y)$ or show that the limit does not exist.

(b) (5 points) Let

$$g(x,y) = \frac{x^2 y}{x^2 + \cos(xy) + y^2}.$$

Either compute $\lim_{(x,y)\to(0,0)} g(x,y)$ or show that the limit does not exist.

8. (a) (5 points) Let $z(x, y)$ be a function of two variables, and suppose that $z(x, y)$ satisfies the equation

$$xyz = \cos(x + y + z).$$

Use implicit differentiation to find z_y. (*Note:* There are some values of x and y for which no z solving the equation exists. Thus, $z(x, y)$ is not defined for all $(x, y) \in \mathbb{R}^2$. Give a formula which is valid under the assumption that $z(x, y)$ is defined.)

(b) (5 points) Let $z(x, y)$ be defined implicitly by

$$F(x, y, z(x, y)) = 0.$$

Derive a formula which expresses z_{xy} in terms of z_x, z_y, and partial derivatives of F. (*Note:* You may assume that $z(x, y)$ is defined for all $(x, y) \in \mathbb{R}^2$, that all first and second order partial derivatives of F and z exist and are continuous, and that F_z is nonzero.)