1. (15 points) Find the equation of the plane that contains the following two lines.

$$\mathbf{r}(t) = \langle 2+t, 2+3t, 3+t \rangle \quad \vec{v}_1 = \langle 1,3,1 \rangle \quad \text{point } (2,2,3)$$
$$\mathbf{s}(t) = \langle 5+t, 15+5t, 10+2t \rangle. \quad \vec{v}_2 = \langle 1,5,2 \rangle$$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} = (6-5)\hat{i} - (2-1)\hat{j} + (5-3)\hat{k}$$
$$= \langle 1, -1, 2 \rangle \rightarrow \text{normal of plane}$$

$$1(x-2) - 1(y-2) + 2(z-3) = 0$$

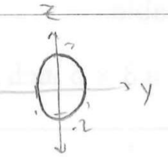
$$\boxed{x - y + 2z = 6}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix}$$

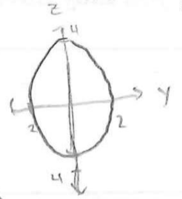
2. (15 points) Describe in words and sketch a picture of the region in $\mathbb{R}^3$ represented by the following inequality. In addition to sketching the region in $\mathbb{R}^3$, sketch the $x = 0$ trace.

$$1 \le y^2 + \frac{z^2}{4} \le 4.$$

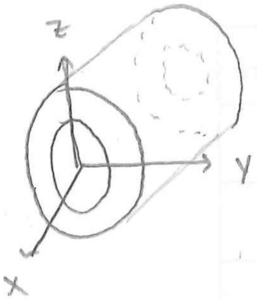
$1 = \frac{y^2}{1} + \frac{z^2}{2^2}$



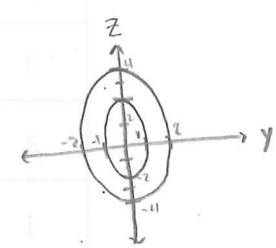
$4 = y^2 + \frac{z^2}{2^2} \rightarrow 1 = \frac{y^2}{4} + \frac{z^2}{16}$



Elliptic cylinder.



$x=0$, $\rightarrow$ yz-plane



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$18+15+15+$
$18+15+10$
$=91$

$\langle 2+1, 3-8, 0-3\rangle$
$\langle 3,-5,-3\rangle$

$\langle 2+1, 5-8, -2-3\rangle$
$\langle 3,-3,-5\rangle$

3. (22 points) Consider the points $P = (-1, 8, 3)$, $Q = (2, 3, 0)$ and $R = (2, 5, -2)$.

(a) The points P, Q, and R form a triangle. Which type of triangle is it? Circle one and justify your answer.

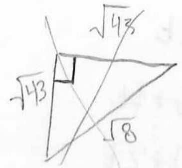
i. Isosceles
~~ii. Right~~
iii. Isosceles and right
~~iv. Equilateral~~
~~v. None of the above~~

$\vec{PQ} = \langle 3,-5,-3\rangle \quad \|\vec{PQ}\| = \sqrt{9+25+9} = \sqrt{43}$

$\vec{QR} = \langle 0, 2, -2\rangle \quad \|\vec{QR}\| = \sqrt{4+4} = \sqrt{8}$

$\vec{RP} = \langle -3, 3, +5\rangle \quad \|\vec{RP}\| = \sqrt{43}$

same length
↳ isosceles
↳ NOT E



→ Not right because $\sqrt{8} < \sqrt{43}$, and $\sqrt{8}$ would be hypotenuse

$$\cos\theta = \frac{\vec{PQ}\cdot\vec{RP}}{\|\vec{PQ}\|\,\|\vec{RP}\|} = \frac{-9-15-15}{43} = \frac{-39}{43}$$ → acute angle, not $90°$

(-1,8,3)

(b) Determine whether the points P, Q, R, and $S = (-1, 9, 5)$ all lie on a plane.

$\vec{PQ} = \langle 3,-5,-3\rangle$

$\vec{PR} = \langle 3,-3,-5\rangle$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & -3 \\ 3 & -3 & -5 \end{vmatrix} = (25-9)\hat{i} - (-15+9)\hat{j} + (-9+15)\hat{k}$$

$\langle 16, 6, 6\rangle = \vec{n}$ normal of plane

$\vec{PS} = \langle 0, 1, 2\rangle$

$\vec{PS}\cdot(\vec{PQ}\times\vec{PR}) = 0$ would mean they all lie on a plane

66
-16
50

$\langle 0,1,2\rangle \cdot \langle 16,6,6\rangle = 6+12 \neq 0$, they do not all lie on a plane

$16(x+1) + 6(y-8) + 6(z-3) = 0$

$16x + 16 + 6y - 48 + 6z - 18 = 0$

$16x + 6y + 6z = 50$

$16(-1) + 6(9) + 6(5) = -16 + 56 + 30 \neq 50$

$$\vec{PS}\cdot(\vec{PQ}\times\vec{PR}) = \begin{vmatrix} 0 & 1 & 2 \\ 3 & -5 & -3 \\ 3 & -3 & -5 \end{vmatrix} = -1(-15+9)\hat{j} + 2(-9+15) = 6\hat{j} + 12 \neq 0$$

/22

$6t^{\frac{1}{2}} \to 3t^{-\frac{1}{2}}$

$\log_e * = y$ $\quad e^y = 1$ ✓ $\quad y = 0$

4. (15 points) Find parametric equations for the tangent line to the curve defined by $\mathbf{r}(t) = \langle 2\ln t, 6\sqrt{t}, t^2\rangle$ at $t = 1$.

$\vec{r}'(t) = \langle \frac{2}{t}, \frac{3}{\sqrt{t}}, 2t \rangle$

$r(1) = \langle 0, 6, 1 \rangle$

$\vec{r}'(1) = \langle 2, 3, 2 \rangle$

$\vec{L}(t) = \langle 0, 6, 1 \rangle + t\langle 2, 3, 2 \rangle$

$= \langle 2t, 6 + 3t, 1 + 2t \rangle$

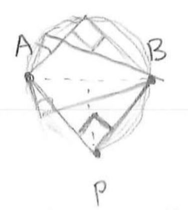
$x = 2t$

$y = 6 + 3t$

$z = 1 + 2t$

5. (15 points) Let $A = (-2, 0, 1)$ and $B = (0, 4, 5)$. Find the set of all points $P = (x, y, z)$ such that $\overrightarrow{AP}$ is orthogonal to $\overrightarrow{BP}$. Give a precise geometric description of your answer.

will give a sphere of points



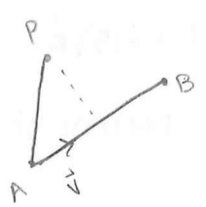
$\overrightarrow{AP} \cdot \overrightarrow{BP} = 0 \qquad P = (P_1, P_2, P_3)$

$\langle P_1 + 2, P_2, P_3 - 1 \rangle \cdot \langle P_1, P_2 - 4, P_3 - 5 \rangle = 0$

$(P_1^2 + 2P_1) + (P_2^2 - 4P_2) + (P_3^2 - 6P_3 + 5) = 0$

$+1 + 4 - 5 + 9$

$(P_1 + 1)^2 + (P_2 - 2)^2 + (P_3 - 3)^2 = 9$



$\vec{v} = \langle 2, 4, 4 \rangle$

$\overrightarrow{AP} = \langle P_1 + 2, P_2, P_3 - 1 \rangle$

Sphere w/ radius 3, and center at $(-1, 2, 3)$

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6. (18 points) Match each vector function with its space curve.

(a) $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ ___C___

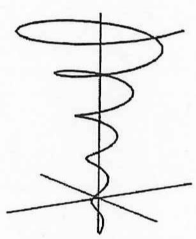
(b) $\mathbf{r}(t) = \langle t, \sin t, \overset{\cos}{\cancel{\sin}}\, t \rangle$ ___D___

(c) $\mathbf{r}(t) = \langle 3 - 2t, 3 - 2t, 1 + t \rangle$ ___F___

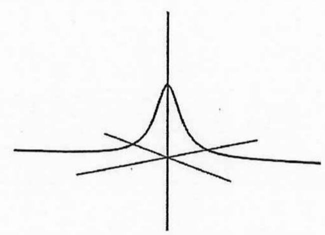
(d) $\mathbf{r}(t) = \langle e^{0.1t} \cos t, e^{0.1t} \sin t, t \rangle$ ___A___

(e) $\mathbf{r}(t) = \langle \cos t, \sin t, \cos(4t) \rangle$ ___E___

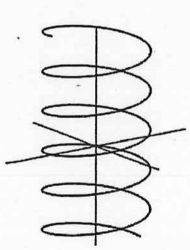
(f) $\mathbf{r}(t) = \left\langle t, -t, \dfrac{1}{1+t^2} \right\rangle$ ___B___



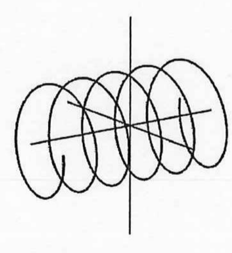
(A)



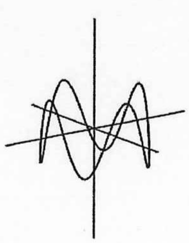
(B)



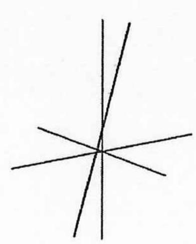
(C)



(D)



(E)



(F)

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