Math 32 A: Multi-Variable Calculus

Fall 2018

University of California, LA

Exam: Midterm 1

Date: Monday 10/22/2018

Time Limit: 40 Minutes

Instructor: Sylvester Eriksson-Bique

Problem	Max Points	Points
1	8	7
2	10	10
3	15	15
4	15	10
5	10	15

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First name: [redacted]

Last name: [redacted]

Student ID: [redacted]

Signature: [illegible]

Date: 10/24/18

By signing here you declare that you will act in accordance with the UCLA Student Conduct Code.

Please circle your discussion section

	Bon-Soon	Eric	Tianqi
Tuesday	1E	1C	1A
Thursday	1F	1D	1B

Policies/Instructions:

- **Remember to write your name and student ID!!**
- No calculators, or books.
- Fit your answer in the space provided. Use the scratch sheets to do calculations before writing.
- Mobile phones turned off and in bags.
- Be considerate; Must remain seated until end of exam.
- Write clearly. You may use the backs of the pages, and the additional last page if needed.
- Check your work. Answer all questions. Make sure you chose between True and False.
- All problems graded on correctness and demonstrating work. **Must show steps and explain to receive full credit.**
- **Good luck!**

Exam continues...

"Success is peace of mind which is a direct result of self-satisfaction in knowing you did your best to become the best you are capable of becoming."

Coach John Wooden

1. (8pt) For each of the following choose True or False (1pt) and **briefly** justify either with picture/computation/words (1pt). Each worth two points.

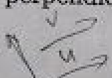
   (a) If $\mathbf{u} = a\mathbf{j} + b\mathbf{k}$ and $\mathbf{v} = c\mathbf{j} + d\mathbf{k}$, then $\mathbf{u} \times \mathbf{v}$ is parallel to $\mathbf{k}$.

   True   False

$$\vec{u} \times \vec{v} = \begin{bmatrix} x & y & z \\ 0i & aj & bk \\ 0i & cj & dk \end{bmatrix} = (ad + bc)ijkx + 0y + 0z$$

= false, as $\vec{u} \times \vec{v}$ has an x component, while not having a z component, which makes it not parallel to $\vec{k}$.

   (b) If $\mathbf{v}$ is parallel to $\mathbf{u}$, and $\mathbf{w}$ is perpendicular to $\mathbf{u}$, then $\mathbf{w}$ is also parallel to $\mathbf{v}$.

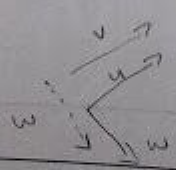


   True   False

It wouldn't be parallel, but rather perpendicular to $\vec{v}$..

$v \parallel u$; $w \perp u$;

$w \perp v$



2. (a) (5pt) Find the area of the triangle with vertices at $P = (1, 0, 2)$, $Q = (0, 1, 2)$ and $R = (3, 2, 1)$.

$\vec{PQ} = \langle -1, 1, 0 \rangle \quad \vec{PR} = \langle 2, 2, -1 \rangle$

A of parallelogram

$$\|\vec{PQ} \times \vec{PR}\| = \begin{bmatrix} x & y & z \\ -1 & 1 & 0 \\ 2 & 2 & -1 \end{bmatrix} = \|x(-1-0) - y(1-0) + z(-2-2)\|$$

$$= \|-x - y - 4z\|$$

$$= \|\langle -1, -1, -4 \rangle\|$$

$$= \sqrt{1^2 + 1^2 + 4^2}$$

$$= \sqrt{18}$$

A of Triangle = $\boxed{\sqrt{18}/2}$

(b) (5pt) Compute the value of the dot product $\mathbf{e} \bullet \mathbf{f}$ and the angle θ between the vectors $\mathbf{e}$ and $\mathbf{f}$, when $\mathbf{f}$ is a unit vector and $||\mathbf{e}|| = 3$ and

$$||\mathbf{f} - \mathbf{e}|| = \sqrt{\cancel{8}\,6}$$

$$\|f - e\|^2 = \|f\|^2 - 2(e \cdot f) + \|e\|^2$$

$$6 = 1 - 2(e \cdot f) + 9$$

$$-4 = -2(e \cdot f)$$

$$\boxed{2 = e \cdot f}$$

$$2 = \|e\|\|f\| \cos\theta$$

$$2 \div 3(1) = \cos\theta$$

$$\frac{2}{3} = \cos\theta$$

$$\boxed{\theta = \cos^{-1}\frac{2}{3}}$$

3. (a) (5pt) Find a unit normal $\mathbf{n}$ to the plane given by the equation $x + y - 2 + 3z = 5$.

$x + y - 2 + 3z = 5$

$x + y + 3z = 7$

$\langle 1, 1, 3 \rangle \cdot x = 7$

$\|\langle 1, 1, 3 \rangle\| = \sqrt{1^2 + 1^2 + 3^2}$

$= \sqrt{11}$

$\vec{n} = \langle \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \rangle$

(b) (5pt) Find the equation of the plane parallel to the previous plane that passes through the point $P = (1, 0, 1)$.

$\langle 1, 1, 3 \rangle \cdot x = \cancel{d}$

$x + y + 3z = d$

$(x-1) + (y-0) + 3(z-1) = d$

$x - 1 + y + 3z - 3 = \cancel{d}$

$x + y + 3z - 4 = 4$

$\boxed{x + y + 3z = 4}$

(c) (5pt) Let $\mathbf{v} = \langle 1, 1, 3 \rangle$. Find the projection

$$\mathbf{v}_{\|\mathbf{n}}$$

of $\mathbf{v}$ onto the normal vector $\mathbf{n}$.

$$V_{\|n} = \frac{\vec{v} \cdot \vec{n}}{\|\vec{n}\|^2} \cdot \vec{n}$$

$$\frac{\langle 1,1,3 \rangle \cdot \langle \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \rangle}{\frac{1}{11} + \frac{1}{11} + \frac{9}{11}} \cdot \langle \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \rangle$$

$$\frac{1}{\sqrt{11}} + \frac{1}{\sqrt{11}} + \frac{9}{\sqrt{11}}$$

$$\frac{\frac{11}{\sqrt{11}}}{1} \cdot \langle \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \rangle$$

$$\langle 1, 1, 3 \rangle$$

4. Let

$r(1)$: 4, 2, 2 $\qquad$ < 5, 2, 0 >

$$\mathbf{r}(t) = \langle 3t + t^2, 2t, 2\rangle \qquad < 3+2t, 2, 0 >$$

(a) (5pt) If $\mathbf{s}(t)$ is another curve with $\mathbf{s}'(1) = \langle 1, 2, 1\rangle$ and $\mathbf{s}(1) = \langle 1, 0, -1\rangle$. Find

$$\frac{d}{dt}(\mathbf{r}\times\mathbf{s})\bigg|_{t=1}.$$

$r(1) = <4, 2, 2>$

$r'(1) = <5, 2, 0>$

$\frac{d}{dt}(r \times s) = r' \times s + r \times s'$

$$\begin{bmatrix} x & y & z \\ 5 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix} + \begin{bmatrix} x & y & z \\ 4 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$x(-2) - y(-5) + (-2z)$

$-2, 5, -2$

$x(2-4) - y(4-2) + z(8-2)$

$-2x - 2y + 6z$

$-2x + 5y + -2z \quad + -2x - 2y + 6z$

$<-2, 5, -2> + <-2, -2, 6>$

$<-4, 3, 4>$

(b) (5pt) If $f(t) = t^2 - 3t + 3$, find $\mathbf{r_2}'(1)$ when $\mathbf{r_2}(t) = \mathbf{r}(f(t))$ is a reparametrization of the above curve.

$r_2'(t) = r'(f(t)) \cdot f'(t)$

$= <3 + 2(t^2 - 3t + 3), 2, 0> \cdot 2t - 3$

$= 3 + 2t^2 - 6t + 6$

$<2t^2 - 6t + 9, 2, 0> \cdot 2t - 3$

$r_2'(1) = <5, 2, 0> \cdot -1$

$= <-5, -2, 0>

5. Let

$$\mathbf{r}(t) = \left\langle \frac{1}{t} + t^2, t+1, \ln(t) \right\rangle$$

be the position vector of a particle traveling through space.

(a) (5pt) Compute the derivative and the second derivative

$$\mathbf{r}'(t), \mathbf{r}''(t).$$

$\frac{2}{t^3}$ $2t^{-3}$

$-t^{-2}$ $2t^{-3}$

$$\boxed{\begin{aligned} r'(t) &= \left\langle -\frac{1}{t^2} + 2t,\ 1,\ \frac{1}{t} \right\rangle \\ r''(t) &= \left\langle \frac{2}{t^3} + 2,\ 0,\ -\frac{1}{t^2} \right\rangle \end{aligned}}$$

(b) (5pt) Find a **unit** tangent vector $\mathbf{T}(t)$ to $\mathbf{r}(t)$ at $t = 1$. Also, find the speed of $\mathbf{r}$ at $t = 1$.

speed $= \|r'(t)\|$

$\|r'(1)\| = \| \langle 1, 1, 1 \rangle \|$

$\boxed{\text{speed} = \sqrt{3}}$

$T(1) = \frac{r'(1)}{\|r'(1)\|} = \frac{\langle 1, 1, 1 \rangle}{\sqrt{3}}$

$\boxed{T(t) = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle}$

(c) (5pt) Find the equation of the tangent line to $\mathbf{r}(t)$ at $t = 1$.

$L(t) = r(t_0) + t(r'(t_0))$

$L(1) = \langle 2, 2, 0 \rangle + t\langle 1, 1, 1 \rangle$

$= \boxed{\langle 2+t, 2+t, t \rangle}$