Midterm 2, Math 31A, Fall 2013
Instructor: Tonći Antunović

Printed name and s

Signed name: ______

Section number, tim

Instructions:

- Read problems very carefully. Please raise your hand if you have questions at any time.
- The correct final answer alone is not sufficient for full credit - try to explain your answers as much as you can. This way you are minimizing the chance of losing points for not explaining details, and maximizing the chance of getting partial credit if you fail to solve the problem completely. However, try to save enough time to seriously attempt to solve all the problems.
- If it's obvious that you final answers can be simplified, please simplify them. Otherwise, your final answers need to be simplified only if this is required in the statement of the problem.
- You are allowed to use only the items necessary for writing. Notes, books, sheets, calculators, phones are not allowed, please put them away. If you need more paper please raise your hand (you can write on the back pages).

Question	Points	Score
1	10	7
2	10	7
3	10	8
4	10	9
5	10	10
Total:	50	41

82%

7

1. (a) (4 points) Write the definition of antiderivative of $f(x)$ on the interval (a, b).

F(x) is an antiderivative of f(x) on the interval (a, b) if $F'(x) = f(x)$ for all $x \in (a, b)$.

4

(b) (2 points) Let $x(t)$ be the concentration of sugar in a solution after t seconds. It changes according to the law

$$x'(t) = \frac{x(t)}{5(x(t)+1)}.$$

$x'(t) = \frac{3}{5(3+1)} = \frac{3}{20}$

If at some point the concentration is $x(t) = 3$ estimate the concentration 2 seconds later (units are not important).

1

At time t when $x(t) = 3$

$t = 0$ $\quad x(t) = 3$ $\quad \frac{3}{20} \times 2$ $\quad x'(2) = x'(t) = \frac{3}{5(3+1)} = \frac{3}{20}$

$x'(t) = \frac{3}{20}$

$x(t+2) \approx x(t) + 2 \cdot x'(t)$

$= 3 + \left(2 \cdot \frac{3}{20}\right) = 3.3$

(c) (2 points) Compute the slope of the tangent line to the curve given by the equation $y \sin y + x \cos y = -1$, at the point $(1, \pi)$. (The point indeed lies on the curve, no need to check that.)

$(1, \pi)$

$(y \cos y\, y' + \sin y) + (x \cdot -\sin y\, y' + \cos y) = -1$

$y \cos y\, y' + \sin y - x \sin y\, y' + \cos y = -1$

$y'(y \cos y - x \sin y) = -1 - \sin y - \cos y$

$y' = \frac{-1 - \sin y - \cos y}{y \cos y - x \sin y}$

0

(d) (2 points) Evaluate the indefinite integral

$$\int (\sin(2x+1) - 2x^2)\, dx.$$

$= \int \sin(2x+1)\, dx - 2 \int x^2\, dx$

$= -\frac{1}{2} \cos(2x+1) - \frac{2}{3} x^3 + C$

2

2. (a) (6 points) Find all local maxima and local minima of the function

$x \neq \frac{5}{9}$ 4

$$f(x) = x^3 - 3x^2 + 9|x - 5|.$$

$-9 - |x+5$
$9(-x-5)$

(You need to find both the locations of local minima/maxima and the function values at local minima/maxima).

$x = ?$
$f(x) = ?$

$f(x) = x^3 - 3x^2 + 9x - 45$
$f'(x) = 3x^2 - 6x + 9$
$x^2 - 2x + 3 = 0$
no roots exist

$f(3) = 18$
$f(-1) = 56$

max. $(-1, 56)$
min. $(3, 18)$? why?

$f(x) = x^3 - 3x^2 - 9x - 45$
$f'(x) = 3x^2 - 6x - 9$
$x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$
$\boxed{x=3}$ $\boxed{x=-1}$

$x = 5$?

(b) (4 points) Find the absolute minimum and the absolute maximum of the same function

$[-2, 0]$ 3

$$f(x) = x^3 - 3x^2 + 9|x - 5|,$$

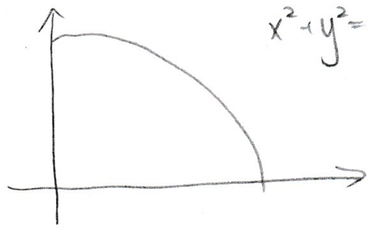
on the interval $[-2, 0]$. (You need to find both the value of the absolute minimum/maximum and the locations of the absolute minimum/maximum).

Compare endpoints to critical points

	x	f(x)
end points	-2	67
	0	45
crit points.	-1	56
	3	18

absolute min. $(3, 18)$
absolute max. $(-2, 67)$

3. (10 points) Consider the arc of the unit circle $x^2 + y^2 = 1$ which lies in the first quadrant (that is $x \geq 0$, $y \geq 0$). Find the coordinates of the point (x, y) on this arc whose sum of the coordinates $x + y$ is maximal possible.



objective function $x+y$

$f(x) = x + y$

$y = \sqrt{1-x^2}$

maximize $f(x) = x + \sqrt{1-x^2}$ on $[0,1]$

$f'(x) = 1 + \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x$

$= 1 - \frac{x}{\sqrt{1-x^2}}$

$f'(x) = 0$ to find critical points

$1 - \frac{x}{\sqrt{1-x^2}} = 0$

$\frac{x}{\sqrt{1-x^2}} = 1$

$x = \sqrt{1-x^2}$

$x^2 = 1 - x^2$

$2x^2 = 1$

$\boxed{x = \sqrt{\frac{1}{2}}}$ $x = -\sqrt{\frac{1}{2}}$ not on interval

Coordinates of point

$x = \sqrt{\frac{1}{2}}$ $y = \sqrt{\frac{1}{2}}$

$y = \sqrt{1 - (\sqrt{\frac{1}{2}})^2}$

$y = \sqrt{1 - (\frac{1}{2})}$

$y = \sqrt{\frac{1}{2}}$

Why max?

$\boxed{x = \frac{1}{\sqrt{2}}}$ $f(\frac{1}{\sqrt{2}}) = \sqrt{2}$

$f(0) = f(1) = 1$

Maximal value $= \sqrt{2}$

Critical points
$x = -1$
$x = 1$
$x = 3$

4. (10 points) Sketch the graph of the function

$$f(x) = \frac{x^2+3}{x-1}.$$

You need to do all the steps in the analysis of the shape of the function.

$$f'(x) = \frac{[(x-1)\cdot 2x] - [(x^2+3)\cdot 1]}{(x-1)^2} = \frac{2x^2-2x-x^2-3}{(x-1)^2} = \frac{x^2-2x-3}{(x-1)^2}$$

Critical points $f'(x)=0$ $x^2-2x-3=0$ $x\ x\ \ -3\ \ 1$ $(x-3)(x+1)=0$ $\boxed{x=3}$ $\boxed{x=-1}$ $\boxed{x=1}$

$x-1 \neq 0$ vertical asymptote at $x=1$

x	$f'(x)$
$(-\infty,-1)$	$+$
$(-1,1)$	$-$
$(1,3)$	$-$
$(3,\infty)$	$+$

$$f''(x) = \frac{[(x-1)^2\cdot(2x-2)] - [x^2-2x-3\cdot 2(x-1)]}{(x-1)^4}$$

$$= \frac{(x-1)^2(2x-2) - 2\cancel{(x-1)}(x^2-2x-3)}{(x-1)^{\cancel{4}3}} = \frac{(x-1)(2x-2)-2(x^2-2x-3)}{(x-1)^3}$$

$$= \frac{\cancel{2x^2}-3x+2\cancel{-2x^2}+4x+6}{(x-1)^3} = \frac{x+8}{(x-1)^3}$$

$f''(x)=0$
$x+8=0$
$\boxed{x=-8}$

$\frac{x^2+3}{x-1}$ $\frac{x^2}{x}$ x

$f''(x)>0$ for $x>1$

graph always concave up

NO

$f''(x)<0$ $x<1$

x	$f''(x)$
$(-\infty,8)$	$+$
$(8,\infty)$	$+$

$x=-1$ max
$x=1$ inflection
$x=3$ min.

$\lim_{x\to 1^-} \frac{x^2+3}{x-1} = -\infty$ $\lim_{x\to\infty} f(x) = \infty$ +4

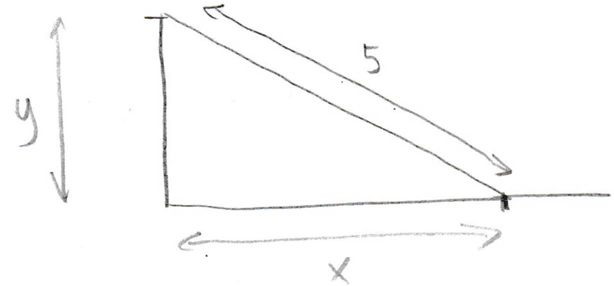
$\lim_{x\to 1^+} \frac{x^2+3}{x-1} = \infty$ +1 $\lim_{x\to -\infty} f(x) = -\infty$

graph on opposite side :)

distance = speed × time

velocity = derivative of distance

5. (10 points) A ladder of length 5 meters is leaned against the wall. The lower end of the ladder (the end touching the ground) is sliding away from the wall at the constant speed of 2 meters per second. Find the velocity of the upper end of the ladder (the end touching the wall) at the moment when the lower end is 3 meters away from the wall.



$$\left.\frac{dy}{dt}\right|_{x=3} = ?$$

$$\frac{dx}{dt} = 2m/s$$

$x = 3$

$y = 4$

$$x^2 + y^2 = 25$$

differentiate

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(3)(2) + 2(4)\frac{dy}{dt} = 0$$

$$12 + 8\frac{dy}{dt} = 0$$

$$8\frac{dy}{dt} = -12$$

$$\frac{dy}{dt} = -\frac{12}{8} = -\frac{3}{2}$$

$y = ?$ $x = 3$

$$y = \sqrt{25 - x^2}$$

$$= \sqrt{25-(3)^2} = \sqrt{16} = 4$$

10