

First Name: _____ ID# _____

Last Name: _____

Rules.

- There are **FOUR** problems; ten points per problem.
- There are extra pages after problems 3 and 4. You may also use the backs of pages.
- No calculators, computers, notes, books, crib-sheets,...
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring, ...
Try to sit still.
- Turn off your cell-phone, pager,...

1	2	3	4	Σ

(1) (a) Define what it means for events A and B to be *conditionally independent* given C .

(b) Suppose $X \sim \text{Poisson}(\lambda)$. What is the probability that $X \leq 2$?

(c) A lock is manufactured with a four-digit code; it is generated randomly with each digit chosen independently and uniformly from $\{0, 1, \dots, 9\}$. What is the probability that the digit 0 appears exactly twice in such a code?

(2) (a) Define a *random variable*.

(b) State the axioms that a *probability law* \mathbb{P} on a sample space Ω must obey.

(c) Show directly from these axioms that $\mathbb{P}(A) \leq \mathbb{P}(B)$ whenever $A \subseteq B$.

- (3) I have three identical bags each containing twelve balls. Suppose one bag has only blue balls, one has half blue and half yellow balls, and one contains three blue balls and nine yellow balls.
- (a) I choose one bag at random and remove one ball from it. What is the probability that the ball is blue?
- (b) Having chosen a bag and removed one ball at random, I find that the ball is blue. What is the probability that I chose the bag containing only blue balls?

extra paper

- (4) Two men and two women line up to buy coffee in a completely random order. Let X denote the number of men who are in front of the first woman in line.
- (a) What is the PMF (probability mass function) for X ?
 - (b) What is $\mathbb{E}(X)$?
 - (c) What is the variance of X .

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