1. (20 points) This problem has parts (a)-(d) Let $w = f(z)$ be the Möbius transform that sends the three points 0,1,-1 to 0,1,∞ respectively.

(a) (5 points) Find an explicit formula for f. Verify that $f(0) = 0$, $f(1) = 1$, and $f(-1) = \infty$.

(b) (5 points) Find the image of $\{z \in \mathbb{C} : |z| < 1\}$ under f.

(c) (5 points) Compute $f^{-1}(i)$.

(d) (5 points) Classify the singularity of f at -1.

2. (15 points) This problem has parts (a)-(c). Consider the function $f = u + iv$, where

$$u(x+iy) = x^2 + 2xy + y^2, \quad v(x+iy) = x^2 - 2xy - y^2.$$

(a) (5 points) Where is f complex differentiable? Where is f analytic?

(b) (5 points) Does there exist an entire function g whose real par is equal to u? If so, give an example of such a function.

(c) (5 points) Does there exist an entire function h whose imaginary part is equal to v? If so give an example of such a function.

3. (15 points) This problem has parts (a)-(c).

(a) (5 points) What are all the possible values of the multi-valued function $\log(i - z)$ can take when $z = 1$?

(b) (5 points) Let $f(z)$ be the principal branch of $\log(i - z)$. Where if f analytic?

(c) (5 points) Find a branch of $\log(i - z)$ which takes the value of $\frac{1}{2}\ln 2 - \frac{5\pi i}{4}$ at $z = 1$.

4. (10 points) Suppose f is an entire funciton and M is a positive number such that $|f(z)| > M$ for all $z \in \mathbb{C}$. Prove that f must be a constant function. (Hint: look at the reciprocal of f.)

5. (20 points) This problem has parts (a) and (b). Let f be the function $f(z) = \frac{1}{z^3(z+1)}$.

(a) (10 points) Find all the Laurent series expansions of $f(z)$ around $z_0 = 0$, and state the regions in which each expansion is valid.

(b) (10 points) Classify the singularities of f in the complex plane, and compute the residues at these singularities.

6. (20 points) This problem has parts (a)-(d).Classify the zero or singularity of hte following funcitons at the indicated point(s). If it is a singularity, then compute its residue.

(a) (5 points) $\frac{\cos z}{2z-\pi}$ at $z = -\frac{\pi}{2}$.

(b) (5 points) $e^{1/z}(1-z)$ at $z = 0$.

(c) (5 points)$\frac{\text{Log}^2 z}{(z-1)e^z}$ at $z = 1$.

(d) (5 points) $\frac{z^2+1}{z^3-z^5}$ at $z = 0$.

7. (10 points) This problem has parts (a), (b) and (c). Let Γ be the contour traversing the circle $|z| = 2$ *twice* in the counter-clockwise direction.

(a) (5 points) Find a parameterization of Γ.

(b) (5 points) Compute $\int_\Gamma |z|\, dz$.

(c) (5 points) Compute $\int_\Gamma \frac{z+e^z}{z(z-4)}\, dz$

8. (15 points) This question has parts (a), (b), and (c). Let Γ_1, Γ_2, and Γ_3 be parameterized by

$$\begin{aligned} z_1(t) &= (1+i)t, \quad 0 \le t \le 2 \\ z_2(t) &= (1-i)t, \quad 0 \le t \le 2 \\ z_3(t) &= 2+it, \quad -2 \le t \le 2 \end{aligned}$$

(a) (5 points) Sketch Γ_1, Γ_2, and Γ_3 in the complex plane.

(b) (5 points) Let f be a continuous function, not identically zero, in the complex plane. Give sufficient conditions to ensure that $\int_{\Gamma_1} f(z)\,dz - \int_{\Gamma_2} f(z)\,dz - \int_{\Gamma_3} f(z)\,dz = 0$.

(c) (5 points) Let $f(z) = \frac{1}{z-1}$. Compute $\int_{\Gamma_1} f(z)\,dz - \int_{\Gamma_2} f(z)\,dz - \int_{\Gamma_3} f(z)\,dz = 0$.

9. (10 points) This problem has parts (a)and(b). Let γ the contour that traverses the line segment from $1 - \frac{i}{2}$ to $1 + \frac{i}{2}$ once in the upward direction.

(a) (5 points) Find a branch of the mulit-valued function $\log(2z - 1)$ which is analytic in a domain tha contains γ.

(b) (5 points) Based on your answer to (a), or otherwise, evaluate the contour integral, $\int_\gamma \frac{2}{2z-1}\, dz$

10. (15 points) Let Γ be the circle $\{z \in \mathbb{C} : |z| = 10\}$, traversed once in the counter-clockwise direction.

(a) (5 points) Compute $\int_\Gamma \frac{e^{z^2}}{(2z-1)^2}\, dz$.

(b) (5 points) Compute $\int_\Gamma \frac{\sin z}{(z-1)(z-3)}\, dz$.

(c) (5 points) Estimate the size of the integral, $\int_\Gamma \frac{e^{i\mathrm{Re}(z)}}{z^2}\, dz$.

11. (15 points) Let f be the power series,

$$f(z) = \sum_{n=0}^{\infty} (n+1) \frac{(z-1)^n}{2^n}$$

(a) (5 points) Find the disk of convergence of this power series.

(b) (5 points) Find a power series expansion for $\frac{2}{3-z}$ around the point $z_0 = 1$. Where does this power series converge?

(c) (5 points) Based on your answer to (b), or otherwise, find a closed form expression for f. (Hint: use differention.)

Math 132 Practice Final Winter 2014

First and Last Name: ______________________ Initial of Last Name: ________

Student ID: ________________

Instructions:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers. If you use the back of a page please indicate that you have work on the reverse side.
- You may not use books, notes, calculators, mobile phones, or any outside help during this exam. You may not collaborate with other students in any way during this exam.

Problem	Points	Score
1	20	
2	15	
3	15	
4	10	
5	20	
6	20	
7	10	
8	15	
9	10	
10	15	
11	15	
Total	165	