Math 132 Midterm
May 1, 2020

Name: _____

Student ID: _____

Instructions: Please complete the exam in 50 minutes and upload to gradescope within 24 hours. Show all of your work and clearly indicate your answers. No computer and calculators are allowed.

1. (20 points) Suppose $f : \mathbb{C} \to \mathbb{C}$ is an analytic function such that $\operatorname{Re} f(z) = -\operatorname{Im} f(z)$ for any $z \in \mathbb{C}$. Show that f must be a constant function.

2. (20 points) Express the following complex numbers in either polar form or standard form:

    (a). $i^{\frac{1}{5}}$.

    (b). $\text{Log}(1+i)$.

    (c). i^{i}.

    (d). 2^{i}.

3. Prove that there are two values of the constant $c \in \mathbb{R}$ such that the function

$$u(x, y) = e^y \cos cx$$

is the real part of an analytic function. Find the analytic function $f(z)$ in each case.

4. (20 points) Compute the following line integral, where the circle γ is counterclockwise oriented.

$$\int_{|z|=4} \frac{z^3 + z + 1}{z^3 - 6z^2 + 5z} dz.$$

5. (20 Points) Given z_1, z_2 two complex numbers lying in the left closed half complex plane, prove that

$$|e^{z_2} - e^{z_1}| \leq |z_2 - z_1|.$$