UCLA

Department of Mathematics

Math 132§3

Spring 2010

Midterm #2

Attempt all problems. As before: although the point total actually adds up to more than 100 you will receive all the points you earn up to 100 points after which you just get 100. Please note that certain problems are *seemingly* impossible but turn out to not too hard with Math 132 techniques. Good luck to one and all, make your answers clear and concise (and EZ to grade). Justify your answers; right answers with little or no derivation will not necessarily receive full credit.

Question #1	/30
Question #2	/20
Question #3	10/20
Question #4	/20
Question #5	/20

Total	/100

Name: ______________ ______________ ____
(Please Print) – Last – – First – M.I.

Student ID Number: ☐☐☐ – ☐☐☐ – ☐☐☐

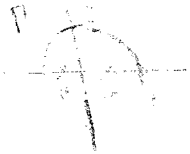
Problem (1) (30 Points) In the following, $f(z)$ is a function that is entire (analytic for all z) while α and β are complex numbers with certain specifications that will be spelled out in each part of the question. We are going to integrate $f(z)[(z-\alpha)(z-\beta)]^{-1}$ around the unit circle ($|z|=1$) in the standard, counterclockwise fashion. In each case, you are going to evaluate the integral expressing your answer in terms of things like α, β, $f(\alpha)$, $f''(\beta)$ etc. In at least one case, the said integral will be ill posed in which case you are to write "problem ill posed". In all cases, a correct answer will recieve full credit. However, you will not receive very much partial credit for a wrong answer with no justification. You may find it helpful to draw some pictures. Watch your factors of $2\pi i$.

[Part A, 6 points.] $|\alpha| > 1$, $|\beta| > 1$.

$$\oint_{|z|=1} \frac{f(z)\,dz}{(z-\alpha)(z-\beta)} = 0$$

[Part B, 6 points.] $|\alpha| < 1$, $|\beta| < 1$.

problem ill posed



[Part C, 6 points.] $|\alpha| < 1$, $|\beta| = 1$.

[Part D, 6 points.] $|\alpha| < 1$, $\beta = \alpha$.

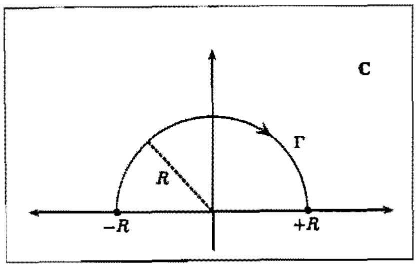
problem ill posed

[Part E, 6 points.] $|\alpha| < 1$, $|\beta| > 1$.

$$\oint_{|z|=1} \frac{f(z)\,dz}{(z-\alpha)(z-\beta)} = 2\pi i\,\frac{f(\alpha)}{\alpha-\beta}$$

Problem (2) (20 Points) Consider the functions

$$P(x,y) = e^x \cos y, \qquad Q(x,y) = e^x \sin y$$



and let Γ denote the semi–circular contour of radius R in the upper half plane starting at $z = -R$ and ending at $z = +R$ which is depicted to the right. Compute the integral

$$\int_\Gamma P dx - Q dy.$$

Problem (3) (20 Points) In the following, we are to construct a function, $f(z)$ which we may call $z^{\frac{1}{4}}$: "$f(z) = z^{\frac{1}{4}}$". Of course there are many candidates for such a function; ours will satisfy the following criteria:

- For $\mathrm{Im}(z) = 0$, $\mathrm{Re}(z) = x > 0$, $f(z) = x^{\frac{1}{4}}$.
- For all z except $\mathrm{Im}(z) > 0$, $f(z)$ is analytic.

Part A (5 points). Provide an unambiguous formula for such a function valid everywhere (except, possibly, the positive imaginary axis).

3

↑ this disc. on pos. real axis

Part B (5 points). For $z = 81$ (i.e. 81 + 0i), compute $f(z)$.

5

Part C (5 points). For $z = -81$ (i.e. -81 + 0i), compute $f(z)$.

Part D (5 points). For $z = -i81$ (i.e. 0 - 81i), compute $f(z)$.

Problem (4) (20 Points) Consider the complex valued function J of the real variable θ given by

$$J(\theta) = \frac{1}{2 + i + e^{i\theta}}.$$

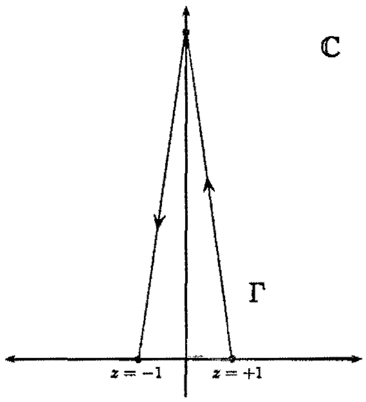
Part A (10 points). Writing $J(\theta) = K(\theta) + iL(\theta)$, find expressions for K and L.

Part B (10 points). Compute $\int_0^{2\pi} K(\theta)d\theta$ where $K(\theta)$ is the function mentioned in Part A above.

Problem (5) (20 Points) Let Γ denote the contour which starts at $z = +1$ goes to $z = 10i$ – along a strait line – and then down to $z = -1$ – along a strait line. This is depicted to the right.

Consider the function e^{-z^2} and show

$$\left|\int_\Gamma e^{-z^2}\,dz\right| \leq 2.$$



M maximum of $|e^{-z^2}|$ along Γ

$M = \frac{1}{e}$

M should be $\frac{1}{\sqrt{101}}$