MATH 132 – SECTION 3
MIDTERM #2

NOVEMBER 20, 2015

Name	
Student ID	

Problem 1	**/25**
Problem 2	**/25**
Problem 3	**/25**
Problem 4	**/25**
Total	**/100**

Problem 1. Prove that a complex polynomial without zeros is constant using Cauchy's Theorem along the following lines. If $P(z)$ is a nonconstant polynomial, write

$$P(z) = P(0) + zQ(z),$$

divide by $zP(z)$, and integrate around an arbitrarily large circle. This will lead to a contradiction if $P(z)$ has no zeros.

Problem 2. Evaluate the following line integral:

$$\oint_{|z-1|=2} \frac{dz}{z^2(z^2-4)e^z}$$

Problem 3. For any complex number α define

$$\binom{\alpha}{n} = \begin{cases} 1 & \text{if } n = 0; \\ \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!} & \text{if } n \geq 1. \end{cases}$$

(a) Find the radius of convergence of the power series

$$(*) \qquad \sum_{n=0}^{\infty} \binom{\alpha}{n} z^n$$

(b) Show that the power series (*) represents the principal branch of the function

$$f(z) = (1+z)^\alpha$$

around $z_0 = 0$.

Problem 4. Find all possible Laurent series expansions centered at $z_0 = 1$ of the function

$$f(z) = \frac{1}{(z-1)(z-2)}$$