MATH 132 – SECTION 3
MIDTERM #1

OCTOBER 23, 2015

Full Name	
Student ID	

Problem 1	**/20**
Problem 2	**/20**
Problem 3	**/30**
Problem 4	**/30**
Total	**/100**

Problem 1. Show that a complex number λ is such that

$$e^{z+\lambda} = e^z$$

for all $z \in \mathbb{C}$ if and only if λ is an integer multiple of $2\pi i$.

Problem 2. Verify the Cauchy–Riemann equations for the real and imaginary parts of $f(z) = z^3$.

Problem 3.

(a) Show that Laplace's equation in polar coordinates is

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0.$$

(b) Show that $\log|z|$ is harmonic on the punctured plane $\mathbb{C}\backslash\{0\}$.

Problem 4. Consider the differential

$$\omega = \frac{-ydx + xdy}{x^2 + y^2}$$

for $(x, y) \neq (0, 0)$.

(a) Show that ω is closed.

(b) Show that ω is *not* independent of path on any annulus centered at $(0, 0)$.