ENG 110 SPRING 2015 MIDTERM SOLUTIONS

Numerical/Analytical Problems:

1. (For this problem, ignore the time value of money!) A company is making plans for constructing an n–story building. The foundation cost, $320,000, is independent of n. The remaining costs will depend on the number of floors, n, indicated by;

TVC(n) = 10,000n² + 60,000n

The price that can be charged for each floor depends on the market demand in the following way;

P = -8,000n+420,000

Additionally, with each floor built, the company will gain expertise that it can use in the following projects. This is worth $72,000 to the company for each floor it constructs.

   a) (5 points) How many floors should there be in the building? (What is optimal n?)

For each floor, the company obtains two different benefits. One is the regular price, P=-8,000n+420,000 and the second is reminiscent of a positive externality, which is the expertise valued at $72,000. Total revenue is given by:

TR(n) = [-8,000n+420,000] n + 72,000n

Marginal revenue is given by:

$$\frac{\partial TR(n)}{\partial n} = -16{,}000n + 420{,}000 + 72{,}000 = -16{,}000n + 492{,}000$$

Marginal cost is given by:

$$\frac{\partial TC(n)}{\partial n} = \frac{\partial TVC(n)}{\partial n} = 20{,}000n + 60{,}000$$

The optimal number of floors, n* is given by:

$$\frac{\partial TR(n)}{\partial n} = \frac{\partial TC(n)}{\partial n}$$

$$-16{,}000n^{*} + 492{,}000 = 20{,}000n^{*} + 60{,}000$$

$$n^{*} = 12$$

b) (3 points) What should be the price for each floor?

P = -8,000*12 + 420,000 = *$324,000*

2. (8 points) Market APR is 12%. We have two options to build a runway in an airport:

Option I has a $5 million initial cost, a $20,000 annual cost and lasts (wears out in) 6 years.
Option II has an $8 million initial cost, a $12,000 annual cost and lasts 10 years.

If the runway will be replaced whenever it wears out, which option should we pick?

We have two ways to go about solving this problem:
1. *Find the common multiples (30 years) and evaluate each project for a 30 year period, calculate the NPV for each and compare them:*
   NPV_I = $5M+$20K (P/A,12%,6) +$5M+$20K (P/A,12%,6)(1/(1.12)^6) +$5M+$20K (P/A,12%,6)(1/(1.12)^{12}) +$5M+$20K (P/A,12%,6)(1/(1.12)^{18}) +$5M+$20K (P/A,12%,6)(1/(1.12)^{24})
   NPV_{II} = $8M+$12K (P/A,12%,10) + $8M+$12K (P/A,12%,10) (1/(1.12)^{10}) + $8M+$12K (P/A,12%,10) (1/(1.12)^{20})

2. *Find the Equivalent Uniform Annual Cost (EUAC) of each option and pick the lower one.*

   $EUAC_I$ = 20,000 + 5M (A/P,12%,6) = $1,236,000
   $EUAC_{II}$ = 12,000 + 8M (A/P,12%,10) = $1,428,000
   Pick the first option.

3. (7 points) Suppose that there are two firms, A and B, each introducing a new product to the market. They will simultaneously decide on whether to charge a high(H), a medium(M), or a low(L) price for their new product, without knowing what the other will charge. The information summarized in the following table is common knowledge:

		Firm B		
		H	**M**	**L**
Firm	**H**	16,40	40,28	36,20
A	**M**	18, 10	38, 50	20,20
	L	10,10	10,30	38,60

For example, if A charges a high price and B charges a medium price, A gets 40 and B gets 28. Given the above payoff structure, under which outcome no firm has any incentive to deviate?

Without loss of generality, start with an action by one firm and iteratively look for the optimal response by the other. For example, given Firm A picks H, what is the best reponse by Firm B?

Firm A	Firm B	Firm A	Firm B	Firm A	Firm B	Firm A	Firm B
H➔	H➔	M➔	M➔	H➔	H➔	M➔	M

After a few iterations, it becomes apparent that we do not converge on a cell ! Try starting with: given Firm A picks L:

Firm A	Firm B	Firm A	Firm B	Firm A	Firm B	Firm A	Firm B
L➔	L➔	L➔	L➔	L➔	L➔	L➔	L

(L,L) is the unique pure strategy Nash Equilibrium. Both firms charge a low price and no firm has an incentive to deviate.

4. (6 points) The diagram below represents a monopolistically competitive firm's cost structure as well as its marginal revenue (MR). You know that at the optimal level of production, this firm is making a loss such that it makes sense to produce in the short run but not in the long run. Given this information, where should the demand be located on the following graph? (Draw the demand on the following graph.)
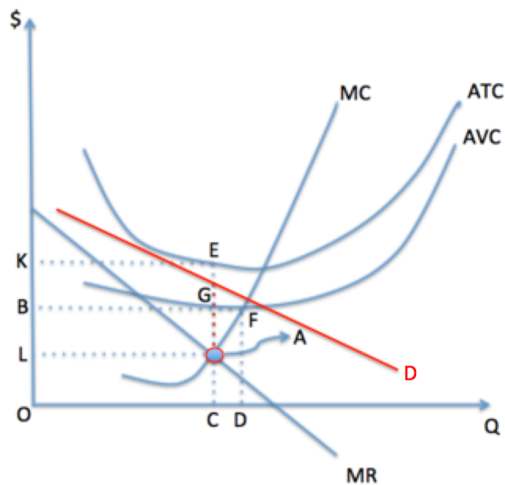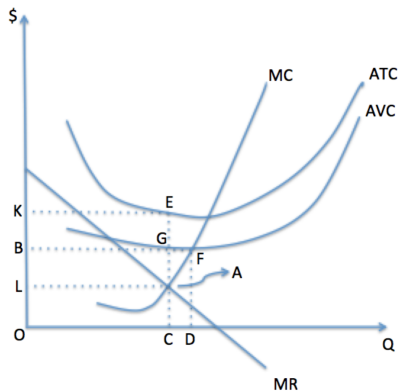
The red line is the DEMAND. It has to be downward sloping and <u>passing through between E and G</u>.

5. (8 points) You started a company five years ago by taking a loan of $100,000. The APR on the loan is 12%. You agreed to make fixed payments every month for 10 years. Today, your 60th payment is due and you decided to make a double payment. (You paid twice what you were paying each month). You will continue to make single payments to pay the rest of the loan. By how many months your loan term shortened? (According to the contract you signed with the bank, you can make double payments, your APR stays the same, your fixed monthly payments do not change, therefore, your loan term shortens.)

Present Value of the Remainder of the debt right after you make your double payment is:

$P_{remainder}$ = *1,430(P/A,1%,60)-1,430 = 1,430x44.955-1,430= $62,855.65*

Now the question becomes: How many months does it take to pay $62,855.65, by paying $1,430 per month at 12% APR?

Table will not be helpful here, we need to apply the formula directly:

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = A(P/A, i, n)$$

$62,855.65=1,430[(1.01)^n-1/0.01(1.01)^n]

$1.01^n=1.7843$,   n=58.19

That is, loan period shortens for about two months.

6. (8 points) You got a loan for $50,000 today at APR 10%. You will pay it in 5 installments in the coming 5 years. The first payment is due exactly a year from today. Each payment is <u>half as much as the previous payment</u>. If at the end of your fifth payment your balance is $0 (you paid the principal as well as all the accumulated interest), how much was your first payment?

The total present value of the payments you make is $5,000.Let's denote the first payment you make by x,

*$50,000=x/1.1 + (x/2)/(1.1)^2 + (x/4)/(1.1)^3 + (x/8)/(1.1)^4 + (x/16)/(1.1)^5$
x=$30,059.*

(One could also solve this problem by applying the <u>geometric uniform series formula</u> which has been introduced in the discussion sessions.)

7. (For this question, ignore time value of money!) A perfectly competitive firm rented two production facilities, A and B, at a total cost of $400. The facilities differ in their productivity levels represented by their respective Total Variable Cost functions.

$TVC_A = 10(Q_A)^2 + 6Q_A$

$TVC_B = 12(Q_B)^2 + 4Q_B$

    a) (5 points) If the price is $100 per Q, what is the profit maximizing level of production? How much of it should be produced in facility A and how much of it should be produced in facility B?

TR(Q_A, Q_B) = 100 (Q_A+Q_B)

Marginal revenue is given by:

$$\frac{\partial TR(Q_A, Q_B)}{\partial Q_A} = 100$$

$$\frac{\partial TR(Q_A, Q_B)}{\partial Q_B} = 100$$

Marginal cost is given by:

$$\frac{\partial TC(Q_A, Q_B)}{\partial Q_A} = \frac{\partial TVC(Q_A, Q_B)}{\partial Q_A} = 20Q_A + 6$$

$$\frac{\partial TC(Q_A, Q_B)}{\partial Q_B} = \frac{\partial TVC(Q_A, Q_B)}{\partial Q_B} = 24Q_B + 4$$

The optimal production level in each facility is given by:

20Q*_A+6=100, Q*_A = 4.7
24Q*_B+4=100, Q*_B = 4

Intuition: *This question is reminiscent if the "stadium problem" we solved during one of our lectures. Allocate the production to a facility as long as the marginal benefit is higher than marginal cost of doing so. If at a certain allocation, marginal cost at facility A is higher than marginal cost at facility B or vice versa, it means that we can reduce the total cost by moving the production from the high marginal cost facility to the low marginal cost facility. At the optimum, it should be the case that, marginal costs are equalized in both facilities.*

b) (3 points) At the optimal level of production, what is the average variable cost (per unit variable cost) for each facility?

$TVC_A = 10(Q_A)^2 + 6Q_A,$ $\quad\quad\quad\quad$ $AVC_A = (10(4.7)^2 + 6*4.7)/4.7 = 53$

$TVC_B = 12(Q_B)^2 + 4Q_B$ $\quad\quad\quad\quad$ $AVC_B = (12(4)^2 + 4*4)/4 = 52$

c) (3 points) Considering your answer in part (b), would you want to use both facilities if the average per unit variable cost is higher in one facility compared to the other? Why or why not?

This question is getting at the heart of the fallacy of thinking with averages rather than marginals when it comes to maximization!

Again, in the spirit of the "stadium problem", average per unit variable cost maybe different at each facility but marginal cost should not be!

Therefore, we would use both facilities even if average variable costs are different at the optimum.

Multiple Choice Questions (4 points each):
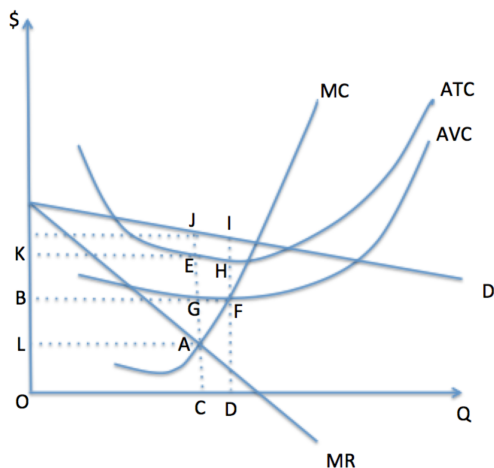
1. For a firm whose cost and revenue structure are represented in the above diagram, which of the following is not correct?

A. Average fixed cost at the profit-maximizing quantity is equal to the difference between K and L.
B. Total Fixed Cost cannot be zero.
C. D is higher than the profit maximizing level of quantity.
D. Marginal cost (MC) crosses ATC at its minimum point.
E. As Q goes up marginal revenue decreases at a higher rate than price does.

2. For a firm whose cost and revenue structure are represented in the above diagram, which of the following represents the Total Variable Cost at the profit maximizing level of output?

A. Area defined by BGLA.
B. Area defined by KECO.
C. Area defined by ALCO.
D. Area defined by BGCO.
E. It cannot be determined with the given information.

3. If a firm reduces its size by a half and its total costs are reduced by more than a half, then:

A. firm is experiencing economies of scale
B. firm is experiencing constant returns to scale
C. firm is experiencing diseconomies of scale
D. firm is expected to go out of business
E. firm would be better of increasing rather than decreasing its scale

4. In economics terms, short run refers to

A. the period during which all raw material going into the production is depleted.
B. the period during which fixed costs cannot be changed.
C. the period during which variable costs cannot be changed.
D. the period during which obtaining positive economic profit is not possible.
E. None of the above.

5. Which of the following is least likely to be a reason for firms to be willing to get bigger?

A. managers' willingness to control bigger firms
B. benefiting from economies of scale
C. to have more control over the price
D. eliminate competition and increase revenue
E. to keep the average variable cost lower than average fixed cost

6. Which of the following statements is correct about a market represented by the following graph?
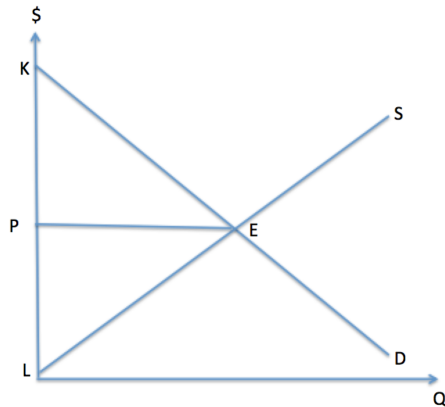
A. Total Consumer Surplus is equal to the area represented by PEL.
B. Total Consumer Surplus is equal to the area represented by KPE.
C. Total Consumer Surplus equal to by the area represented by KLE.
D. Total Consumer Surplus is equal to the area represented by KLE minus total revenue.
E. None of the above.

Current Event Questions (4 points each):

The following questions are drawn from the class discussions on current events. Please give a short answer for each (2 to 3 sentences) within the context they were discussed in class.

1. Major oil producers of the world are finding themselves in a challenging marketplace due to increasing production via fracking and horizontal drilling undertaken by relatively smaller companies. Oil supply has been increasing leading to a price decline in the market. (per barrel oil fell from $105/barrel to as low as $40/barrel in less than a year.)

Why would a declining price be strategically beneficial to major oil producers?

This example was discussed within "be a monopoly and/or stay a monopoly" topic.
Low prices maybe hurtful to the major oil producers in the short run, however, it is more hurtful to small producers that have higher marginal costs.

In the long run, low prices can eliminate existing competition. Further, they provide a disincentive to enter by others.

2. Google faces antitrust inquiries in Europe where it holds 92% of the online search market. Why are European authorities going after Google?

This example was discussed within "monopoly as a market structure" topic.

Being a monopoly, that is, having a high market share does not constitute grounds for antitrust violations, but engaging in anti-competitive behavior does. It is claimed that Google is bundling its service websites with its search algorithm and displays its own

websites above other companies' websites thereby distorting rankings of search query results.

3. Freemium business model is used by an increasing number of online companies. What is freemium business model? What are the <u>two</u> ways in which it increases companies' profits?

This example was discussed within the "price discrimination" topic.
Freemium model refers to offering a basic version of a service or a product free-of-charge and charging for a premium version. Two ways in which it can be profitable:

1. *Differentiates high-willingness-to-pay customers and low-willingness-to-pay customers.*

2. *The valuation of each customer is further increased by increased usage by others (network externalities).*

4. Whatsapp, which is recently acquired by Facebook for $22 billion, is an instant messaging app for smartphones. Other than eliminating a potential competitor, what did Facebook get by purchasing Whatsapp?

This example was discussed within the "online revenue streams" topic.

Information that is exchanged via text messages were closed to online companies since they were taking place within mobile service platforms. By bringing it a online platform, Whatsapp made it possible to tap into consumer information. Therefore, Facebook purchased "current and future consumer profile/preference data".

5. Suppose a company produces two products, A and B, and sells them by bundling. That is, company sets a price for the pair and sells A and B as a pair. What does the company achieve by bundling? Why not sell each product separately?

Bundling is a form of price discrimination! It allows to extract more from high-willingness-to-pay customers while also obtaining revenue from low-willingness-to-pay customers.

BONUS(2 points): We post various articles from well-established online news sources such as NYTimes and The Economist. You can read these articles without having to subscribe to the respective sites. Which pricing practice allows you to do that?

Price discrimination

I. Cost Structure and Estimation: $\text{TC=TVC+TFC}, \quad \text{ATC} = \text{TC/Q}, \quad \text{MC} = \Delta\text{TC}/\Delta\text{Q}$

II. Single Payment:

$F = P(1+i)^n \qquad F = P(F/P, i, n) \qquad P = F(1+i)^{-n} \qquad P = F(P/F, i, n)$

III. Effective Rate: $i_a = (1+\dfrac{r}{m})^m - 1$

IV. Uniform Series:

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] = A(F/A, i, n) \qquad A = F\left[\frac{i}{(1+i)^n - 1}\right] = A(A/F, i, n)$$

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = P(A/P, i, n) \qquad P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = A(P/A, i, n)$$

Arithmetic Gradient Present Worth Factor:

$$P = G\left[\frac{(1+i)^n - i \cdot n - 1}{i^2(1+i)^n}\right] = G(P/G, i, n)$$

Arithmetic Gradient Uniform Series Factor:

$$A = G\left[\frac{(1+i)^n - i \cdot n - 1}{i(1+i)^n - i}\right]$$

$$= G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right] = G(A/G, i, n)$$

Geometric Gradient Present Worth Factor:

$$P = A_1\left[\frac{1 - (1+g)^n(1+i)^{-n}}{i - g}\right] \quad \text{where} \quad i \neq g \qquad\qquad P = \frac{n \cdot A_1}{(1+i)} \quad \text{where} \quad i = g$$

Capitalized Cost: $A = Pi$