

**NUMERIC QUESTIONS:**

1. (10 points) A firm has a cost of borrowing of 9%, WACC of 11%, and an opportunity cost of 12%. Firm has two options to choose from regarding its energy costs going forward (energy costs in the foreseeable future). Option 1 has a \$120,000 initial cost and a 17-year lifetime. Option 2 has a \$140,000 initial cost and 35-year lifetime. Which option is the least costly option for the firm taking into account the time value of money?

*The firm will use a rate of  $\max\{9\%, 11\%, 12\%\} = 12\%$  as its discount rate.*

*Once an option is picked, it will be used for the foreseeable future. Therefore, the option will be repeated when its lifetime is over. Then, to be able compare two options,*

*EITHER we need to calculate the NPV for each that takes into account all the cash flow during least common multiple of 17 and 37 years, which is  $17 \times 37 = 629$  years.*

*OR, for each option, we need to calculate the EUAC.*

*Option 1:  $120,000 = A (P/A, 12\%, 17)$  use the table to find  $A = \$16,854$*

*Alternatively, use the annuity formula:  $120,000 = A (1/0.12) (1 - 1/1.12^{17}) = \$16,855$  (difference is due to rounding)*

*Option 2:  $140,000 = A (P/A, 12\%, 37)$  use the table to find  $A = \$17,123$*

*Alternatively, use the annuity formula:  $140,000 = A (1/0.12) (1 - 1/1.12^{37}) = \$17,124$  (difference is due to rounding)*

*OPTION 1 is the least costly one.*

2. Tayda Inc. is operating in a monopolistically competitive market where industry demand is given by  $P = 120,000 - 60Q$ . Tayda's demand is  $1/20^{\text{th}}$  of the industry demand at each price and it has a total cost given by  $TC(Q) = 2,500,000 + 4Q^2$ . All costs are due today and Tayda will get all revenue in one year (demand will stay as it is). Market rate is 10%.

a) (12 points) What is the Internal Rate of Return of Tayda's business?

*Tayda has a marginal cost of  $MC(Q) = 8Q$ .*

*Given each  $P$ , Tayda faces  $1/20^{\text{th}}$  of the industry demand, therefore Tayda's demand can be obtained as  $Q = \{(120,000 - P)/60\}/20$ , which is equal to  $P = 120,000 - 1,200Q$ .*

*Graders: If the student DID NOT divide the demand by 20, but used industry demand instead, take off 5 points.*

*The marginal revenue that corresponds to this demand line is  $MR = 120,000 - 2,400Q$ . (Find total revenue and take its first derivative with respect to  $Q$ ). As the revenue will be obtained in one year, the present value of the marginal revenue must be used in determining the optimal quantity to be produced.*

$$MC(Q) = MR(Q)/1.1$$

$$8Q = (120,000 - 2,400Q) / 1.1, Q = 120,000 / 2,408.8 = 49.82$$

Total Cost, which is due today, is given by:

$$TC(Q) = 2,500,000 + 4Q^2$$

$$TC(49.82) = 2,500,000 + 4*49.82^2 = 2,509,927$$

Total Revenue, which will be obtained in one year, is given by:

$$TR(Q) = P(Q)*Q$$

$$TR(49.82) = (120,000 - 1,200*49.82)*49.82 = 60,219.2*49.82 = 2,999,960$$

To find internal rate of return, we need to equate NPV of this cash flow to zero:

$$NPV = -2,509,927 + 2,999,960/(1+IRR)$$

$$IRR = 19.52\%$$

- b) (5 points) If Tayda can perfectly price discriminate at the optimal Q obtained in part (a) what would be the Internal Rate of Return of Tayda's business?

The extra revenue to be obtained would be the area of a triangle under Tayda's demand defined by a base of 49.82 and a height of (120,000-60,219.2) :

$$(120,000 - 60,219.2)*49.82/2 = 1,489,140$$

Then, to find internal rate of return, we need to equate NPV of the new cash flow to zero:

$$NPV = -2,509,927 + (2,999,960 + 1,489,140)/(1+IRR)$$

$$IRR = 78.85\%$$

3. You are considering getting a loan from a financial institution to pay back in 3 years. You will be charged 10% per year and you are offered two payment plans to choose from:

Plan I: Pay everything at the end of 3 years.

Plan II: Pay the accumulated interest at the end of each year and the principal at the end of 3 years.

- a) (2 points) If you can make 8% in the market which plan would you choose?

CHOOSE PLAN II. Since you make less money than the rate your debt grows at, you should pick the option that will allow you to pay sooner.

- b) (2 points) If you can make 10% in the market which plan would you choose?

You are indifferent. Either Plan I or Plan II.

- c) (2 points) If you can make 12% in the market which plan would you choose?

CHOOSE PLAN I. Since you make more money than the rate your debt grows at, you should pick the option that will allow you to pay later.

4. You got a loan for \$1,000 today and will make 6 annual payments beginning a year from today. Bank charges you 4% annually (annual compounding).

- a) (8 points) If each of your payments is twice as big as the previous one, how much did you pay in year 1?

Let your first payment be  $x$ . Then the present value of all your payments should be equal to \$1,000.

$$1,000 = x/1.04 + 2x/1.04^2 + 4x/1.04^3 + 8x/1.04^4 + 16x/1.04^5 + 32x/1.04^6$$

$$x = \$19.36$$

- b) (5 points) If apart from the payments calculated in part (a) bank charges \$42 each year, what is the internal rate of return of your loan? (Writing the equation to solve for IRR is enough to get full points. You do not need to solve for IRR.)

$$NPV = -1,000 + (19.36+42)/(1+IRR) + (38.72+42)/(1+IRR)^2 + (77.44+42)/(1+IRR)^3 + (154.88+42)/(1+IRR)^4 + (309.76+42)/(1+IRR)^5 + (619.52+42)/(1+IRR)^6$$

- c) (10 points) Before your payments begin, bank offers to eliminate \$42 fee if you make your payments calculated at 8% rather than 4% annually. Would you take this deal?

Let your new first payment be  $y$ . Then the present value of all your payments should be equal to \$1,000.

$$1,000 = y/1.08 + 2y/1.08^2 + 4y/1.08^3 + 8y/1.08^4 + 16y/1.08^5 + 32y/1.08^6$$

$$y = \$23.39$$

Then you pay an extra of 4.03 in the first year compared to before. That is  $y-x = 4.03$ .

Then you need to compare paying \$42 extra each year versus paying extra \$4.03 in the first year and doubling it progressively. The cash flow you need to compare is given below:

Year	Extra payment if you take first option	Extra payment if you take second option	EUAC of second option
1	42	4.03	37.27
2	42	8.06	37.27
3	42	16.12	37.27
4	42	32.24	37.27
5	42	64.48	37.27
6	42	128.96	37.27

Therefore, you should pick the second option.

Graders: If the student calculated  $y$  but did not calculate EUAC but said something along the lines of: "With extra \$42 you are paying **more upfront** compared to the second option of paying everything at 8%" give 7 points.

5. (6 points) Today is the first day of year 1. What is the Net Present Value of a project that demands \$10,000 today and pays back \$4,000 per year 3 times beginning from a year from today if your best opportunity is 10% in the first year, 12% in the second year and 14% in the third year?

$$NPV = -10,000 + 4,000/1.1 + 4,000/(1.1)(1.12) + 4,000/(1.1)(1.12)(1.14)$$
$$NPV = -268.85$$

*If the student calculated instead:*

$$-10,000 + 4,000/1.1 + 4,000/(1.12)^2 + 4,000/(1.14)^3$$

*Take off 3 points.*

6. (8 points) Jill invests \$1,000 for 4 years at 12% annual rate compounded quarterly for the first 2 years and compounded monthly the following 2 years, what is the effective rate she received per year?

*If instead, Jill were to invest \$1,000 at x% compounded annually for 4 years, and obtain the same amount at year 4, then the following equation would be satisfied:*

$$1,000(1+3\%)^8(1+1\%)^{24} = 1,000(1+x)^4$$

$$x = 12.62\%$$

*Graders: If the student calculated instead:*

$$\text{Effective rate for first two years: } (1+3\%)^4 - 1 = 12.55\%$$

$$\text{Effective rate for the last two years: } (1+1\%)^{12} - 1 = 12.68\%$$

$$\text{Overall Effective Rate} = (.1255 * .1268)^{1/2} = 12.62\% \text{ GIVE FULL POINTS...close enough in terms of the method.}$$

$$\text{But instead of the last step if the student just took the average } (12.55\% + 12.68\%) / 2 = 12.62\%$$

*TAKE OFF 2 POINTS. This is a good approximation but not exactly equal to the effective rate since taking an average does not take into account the compounding!*

**TRUE/FALSE QUESTIONS (5 points each): Please explain your reasoning with a few sentences. NO POINTS will be given if no explanation is provided!**

7. TRUE/FALSE: Aluminum Industry cannot be characterized as an oligopoly since aluminum is a homogenous good.

*FALSE: An oligopolistic market structure can have heterogenous (differentiated commodities) as well as homogenous ones. (Monopolistic competition is the market structure that is characterized by product differentiation!)*

8. TRUE/FALSE: Masha invests \$1,000 at 10% per year. Masha would prefer quarterly compounding to annual compounding, and further prefer monthly compounding to quarterly compounding.

*TRUE: As the frequency of compounding increases, accumulated interest goes up.*

9. TRUE/FALSE: In light of the previous question, extra money that could be made by more frequent compounding will increase at an increasing rate.

*FALSE: As the frequency of compounding goes up, the extra interest that accumulates goes down. For example, as the compounding frequency increases and goes to infinity, the total accumulated interest would be finite (that is, it converges to a finite number). If the above statement were to be true, this would not be the case and the accumulated interest would be infinite.*

**10. TRUE/FALSE:** Natural Monopoly emerges in a market where the total demand is too low compared to the average total cost of the production.

*TRUE. In a market where demand is too low compared to the average total cost (that is demand barely covers costs) no two firms can share this demand and profitably operate. Then, naturally, there can be only one firm that would profitably operate in such a market, hence the name "natural monopoly".*

**11. TRUE/FALSE:** It is not possible to price discriminate and not advised to advertise under perfect competition since the commodity is homogenous (that is, every firm is selling the same commodity with no differentiation.)

*FALSE. It is not advisable to advertise under perfect competition since there is no product differentiation. However, companies can still price discriminate since price discrimination refers to charging different prices to different customers for the same commodity.*

**12. TRUE/FALSE:** You are offered two mutually exclusive projects: A and B. A has a lending type cash flow. Each has cash flow that changes sign only once from year 0 to year 1. If you make the correct decision by choosing B when A-B has 5% IRR and the market rate is 7%, then B must have a positive cash flow in year 3.

*It is deduced from the given information that A as well as A-B has lending type cash flows (-,+ ,+ ,+...). However, B can still have a lending type or a borrowing type cash flow. Therefore, B does not necessarily have a positive cash flow in year 3.*

**I. Cost Structure:**  $TC=TVC+TFC$ ,  $ATC = TC/Q$ ,  $MC= \Delta TC / \Delta Q$

**II. Single Payment:**

$$F = P(1+i)^n \quad F = P(F/P, i, n) \quad P = F(1+i)^{-n} \quad P = F(P/F, i, n)$$

**III. Effective Rate:**  $i_a = (1 + \frac{r}{m})^m - 1$

**IV. Uniform Series(Annuity):**

$$F = A \left[ \frac{(1+i)^n - 1}{i} \right] = A(F/A, i, n) \quad A = F \left[ \frac{i}{(1+i)^n - 1} \right] = F(A/F, i, n)$$

$$A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] = P(A/P, i, n) \quad P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] = A(P/A, i, n)$$

**Perpetuity:**  $A = Pi$