ENG 110
Fall 2017 **Second Midterm Solutions** **Mel Bulu-Taciroglu**

1. (14 points) How long will it take for an investment to triple at 4% APR compounded annually if half of the interest accumulated within the previous year is withdrawn at the end of each year?

This question is the same as the following question: "How long will it take for an investment to triple at 2% APR compounded annually?"

In order to see this shortcut we can write down the progression of the investment year by year. Suppose $x is invested at 4% annually.
At the end of the first year, total accumulation will be:

$$x(1+0.04)-\frac{0.04x}{2}=x(1+0.02)$$

At the end of the second year total accumulation is:

$$x(1+0.02)(1+0.04)-\frac{0.04\,x(1+0.02)}{2}=x(1+0.02)^2$$

At the end of the n^{th} year, the total accumulation is $x(1+0.02)^n$.
Then $x(1+0.02)^n=3x$ ➔ n=ln3/ln1.02 = 55.48 years.

2. There is a popular new product in the market, likely to attract multiple producers into the market. If you start production today, you will need $24,000 for fixed costs and variable costs will be $TVC(Q) = 2Q^2+5Q$. All costs are due when the production starts. You do not have any funds today but you will have funds in exactly a year from today (enough to cover all costs). You can get $700 in today's dollars per unit if you start production today (year 0). If you start the production a year from today (year 1), you will get $560 in year 1 dollars but you will have enough time to make necessary adjustments to have your variable costs go down by 20% and fixed costs go down to $14,000.

If you can lend at 8% per year and borrow at 6% per year, would you start production today or a year from today?

Explanation:

Note that this is a variation on one of your recent homework questions. As your market (discount) rate, you need to use the maximum of your borrowing and opportunity costs. Your borrowing and lending rates are NOT the same, but still, it does not matter if you have funds now or not. If you borrow, you will need to pay 6% for every dollar you borrow. Instead of investing on the production, you can directly make 8% by lending it. Therefore, at least 8% needs to be recouped from the production for it to be preferable to directly lending the money. On the other hand, if you use your own funds, the opportunity cost is 8%, that is, each dollar needs to make 8%. Either out of pocket or borrowed, every dollar used in this business needs to earn 8% before it starts to turn in any ***positive economic*** *profit.*

Solution: To be able to compare the value of starting production now versus a year later, all numbers should be brought to the same time period. The following solution assumes bringing each value to today (year 0). (Graders, note that another solution based on another reference point might be given in students' answers)

If we start the production today: P=MC, 700 = 4Q+5, Q=173.75

*Total Profit today = 700*173.75-24,000-2*173.75^2-5*173.75 = 36,378*

If we start the production a year from today (year 1): P=MC, 560 = 0.8(4Q+5), Q=173.75

*Total Profit in year 1 = 560*173.75-14,000-0.8*(2*173.75^2+5*173.75) = 34,303*

Total Profit today = 34,303/1.08 = 32,361 (Graders: if this step is skipped, take off 8 points!!!)

Start production today.

3. Jules starts a company and negotiates with an investor who offers the following two deals:

A. Get $100,000 today and pay back $120,000 a year from today.

B. Get $200,000 today and beginning a year from today, pay 5% of your profit every year (assume you expect to make $500,000 per year for the foreseeable future)

a) What is the internal rate of return of taking deal B?
b) Prove that NPV and IRR leads you to choose the same alternative if the market rate is 21%.

Solution:

a) (7 points) Cash flow for the deal A is +100k and -120k, on the other hand, cash flow for deal B is +200k, -25k, -25k, -25k, -25k,…
The incremental cash flow of B over A is +100k, +95k, -25k, -25k….
Setting the NPV equal to zero and solving for r will give us the internal rate of return of deal B.

+100k +120k/(1+r) - 25k/r = 0

100r^2+195r-25 = 0

r=12.07%

Graders: Answer thus far gets the full points.

Interpretation: For every dollar borrowed over and above deal A, we pay on average 12.07% per year. If we can borrow in the market at a lower rate than 12.07, then B is a bad deal. If the borrowing rate in the market is above 12.07%, then deal B is a good one.

b) (7 points) IRR Method: If the market rate is 21%, borrowing at 12.07% is a good deal. Then, B is preferred to A.
NPV method:

+100,000-120,000/1.21= 826.44
+200,000-25,000/1.21-25000/1.21^2+…. = 200,000-25,000/0.21=80,952

Deal B is preferred!

4. Lisa can get a basic model of iPhone for $600 without having to sign up with a carrier. If she signs a contract with Verizon, she can get it for $200. To make up for the difference, Verizon requires Lisa to make constant monthly payments for 2 years (total of 24 payments) starting from a month from the purchase date. At the time of the purchase, annual interest rate is 6%. After a year, Verizon increases the constant monthly payment for the rest of the contract due to an increase in the interest rates from 6% to 9%.

By how much should the constant monthly payment be increased so that Verizon exactly makes up for the discounted iPhone price?

Let A_1 be the monthly payment for the first year and A_2 be the monthly payment for the second year.

A_1=400(A_1/P, 6%/12, 24) = 17.73. (4 points)

After the first year, what is left to be paid is P=$A_1\left(\frac{P}{A_1},\frac{1}{2}\%,12\right)$. This can also be calculated as P 400(1+0.5%)^{12} - A_1(F/A_1, 0.5%,12). P = 205.98. (5 points)

Then $A_2 = P(\frac{A_1}{P},\frac{3}{4}\%,12)$ = 18.01.(5 points)

5. The CROC Co. is considering a new milling machine. They have narrowed the choices down to three alternatives in addition to the Null (Do nothing) alternative. The relevant data are shown in the table below.

	A	B	C
First Cost	$75,000	$125,000	$220,000
Annual Benefit	$28,000	$43,000	$79,000
Annual Costs	$8,000	$13,000	$38,000
Salvage Value	$3,000	$6,900	$16,000

All machines have a life of ten years. Using **incremental rate of return analysis**, which alternative should the company choose? Use a MARR of 15%.

Explanation: As we discussed many times in class, solving an internal rate of return problem involves higher order polynomials analytically, and time-consuming calculations for trial and error numerically. Therefore, an alternative method is called for. As posted under WEEK 6 lecture notes, the relation between NPV and IRR could be employed in order to solve the problem using **incremental IRR analysis.** *Note that full points can be obtained only by solving the problem with* **incremental IRR**.

Order the options in increasing First Costs: A, B, and C.

If only NPVs calculated but no mention of IRR then take off 12 points.

Math error: take off 1 point.
If the math error is an indication of a conceptual error, more points will be taken off (case-by-case basis.)

Increment (Null up to A): If student did not compare A to null take off 5 points.

NPW = -75,000 + {(28,000 - 8,000) (P/A, i, 10)} + {(3,000(P/F, i, 10)}.
= -75,000 + {20,000(P/A, 15%, 10)} + {3,000(P/F, 15%, 10)} = $26,121.60.

Since, NPW is greater than $0, incremental IRR(Internal Rate of Return) is greater than MARR (since only a higher rate would carry NPW to zero).
Accept A.

Increment (B to A):

NPW = (125,000 - 75,000) + [{(43,000 - 13,000) - (28,000 - 8,000)} (P/A, i, 10)] +
{(6,900 - 3,000) (P/F, i, 10)}.
= -50,000 + {10,000 (P/A, 15%, 10)} + {(3,900(P/F, 15%, 10)} = $1,154.08.

Since, NPW is greater than $0 incremental IRR is greater than MARR.
Accept B.

Increment (C to B):

NPW = - (220,000 - 125,000) + [{(79,000 - 38,000) - (43,000 - 13,000)} (P/A, i, 10)] +
{(16,000 - 6,900) (P/F, i, 10)}.
= -95,000 + {11,000(P/A, 15%, 10)} + {(9,100 (P/F, 15%, 10)} = -$37,541.48.
Since, NPW is less than $0, incremental IRR is less than MARR.
Reject C.

B should be chosen.

6. LAX runway pavement is replaced every two years at a cost of $1million. The most recent replacement happened exactly a year ago. There is a new construction material that will make the pavement last twice as long. California Department of Transportation is considering to switch to this new material and make the replacement TODAY. What is the maximum amount to be paid for this new material if the market rate is 10%?

Since the lifetimes are not equal and projects will be repeated, NPV cannot be used. We should employ EUAW.

$EUAW_{old}$: old material:

$\$3million = EUAW_{old} + EUAW_{old} / 1.1$

$EUAW_{old} = \$1.5714million$ (5 points)

If the old pavement completed its lifetime today, the max of the new material would be equal to the present value of 4 $EUAW_{old.}$, which is $1.5714+1.5714/1.1+1.5714/1.1^2+1.5714/1.1^4 = 1.5714+1.5714$ (P/A,10%,3) = 5.4792. (5 points)

However, by adopting the new material today and replacing the pavement we are entering the process before the last pavement completes its lifetime, that is, we are wasting 1 year's-worth of old pavement. Then, the total cost of new pavement should be lower than the above calculated amount for this switch to make economic sense. But, by how much lower?

*By switching to the new material, we will waste $1.5714million today. Then, this amount should be distributed to the entire lifetime of the new pavement. Using perpetuity formula, A=Pi, A=1.5714*0.1=0.15714.*

Therefore, new material should cost this much less per year, 1.5714-0.15714 = 1.4143. Over its one lifetime (4 years), the new material's cost should be

$1.4143+1.4143/1.1+1.4143/1.1^2+1.4143/1.1^4 = 1.4143+1.4143\ (P/A,10\%,3) = 4.9314.$ *(4 points)*

7. a) Suppose UCLA either charges a one-time, fixed tuition of $40,000 at the start of the school OR asks for 3% of the salary after graduation (starting exactly 4 years from the entrance). If you are expected to work for 30 years after graduation and your MARR is 4%, what is your LEAST expected annual salary if you choose to pay $40,000 upfront?

Let your annual salary be x. Then, $40{,}000<0.3x\ (P/A,4\%,30)\ (1/1.04^3)$

$x = 86{,}735$ *(7 points)*

b) Suppose your expected salary is 20% more than the amount you found in part (a). What is the internal rate of return of paying your tuition upfront? (You are expected to write the equation that will give this IRR. You are NOT expected to solve for IRR numerically!)

$1.2*86{,}735 * 0.03 = \$3{,}122$

$-40{,}000+ 3{,}122/(1+r)^4+3{,}122/(1+r)^5+3{,}122/(1+r)^6+3{,}122/(1+r)^7+\ldots+3{,}122/(1+r)^{32}+3{,}122/(1+r)^{33} = 0$ *(7 points)*