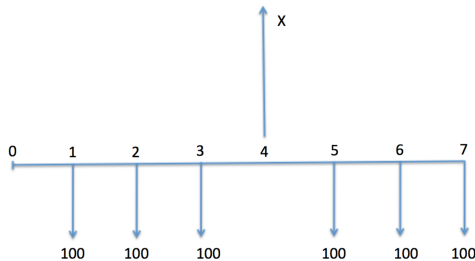


Multiple Choice Questions (6 points each)

1. What is the “opportunity cost” within the context of determining WACC?

- A. The rate of return on the least desirable project
- B. The rate of return on the most promising project
- C. The rate of return on the best rejected project
- D. The desired rate of return
- E. The minimum rate of return of the rejected projects

2. Given the cash flow diagram below, the unknown value X may be computed from the following equation:



- A. $[100(P/A, i, 6) - 100(P/F, i, 4)] [(F/P, i, 4)]$
- B. $[100(F/A, i, 3) + 100(P/A, i, 2)]$
- C. $[100(F/A, i, 4) + 100(P/A, i, 3)] - 100$
- D. $[100(P/A, i, 6) + 100(F/P, i, 2)] [(P/F, i, 2)]$
- E. None of the above

3. You and your friend just quit Google where you were making \$120,000/year each. A third friend offers you a job at her company for \$130,000 each but you both refuse to start a company of your own that will produce customized cell towers. You rent an office space for \$40,000 per year, get raw materials for \$60,000 /year, and spend \$80,000 on other administrative, marketing and production costs. What was your total cost for your first year?

- A. \$420,000
- B. \$300,000
- C. \$180,000
- D. \$440,000
- E. \$400,000

$$60,000 + 80,000 + 40,000 + 130,000 + 130,000 = \$440,000$$

4. You invest \$50,000 today for two years. First year, you get 12% with annual compounding and second year you get 12% with monthly compounding. You get back your investment (principal and total interest accumulated) at the end of the second year. What was the annual internal rate of return of your investment?

- a) Less than 12%
- b) 11.9%
- c) 12.34%
- d) 12%
- e) None of the above

1st method: $50,000 * (1.12) * (1.01)^{12} = 50,000(1+r)^2$

2nd method: $-50,000 + [50,000 * (1.12) * (1.01)^{12}] / (1+r)^2 = 0$

$r = 12.34\%$

5. You are presented with two mutually exclusive projects, A and B. You end up making the right decision by accepting project A when the IRR of B-A is 8 % while the market rate is 10%. Then, you can be sure that:

- A. the initial cash flow of A is positive.
- B. the initial cash flow of B is positive.
- C. the initial cash flow of A is negative.
- D. the initial cash flow of B is negative.
- E. the initial cash flow of A-B is positive.

6. You have three options, A, B, and, C, in the order of increasing initial costs, to install solar panels on the roof of your factory. The relevant data are shown in the table. Using incremental rate of return analysis, which alternative should you choose if MARR is 15%.

	IRR
A	12%
B	18%
C-B	16 %

- A. A
- B. B
- C. C
- D. None of the above
- E. Cannot be determined with given data

The projects have initial costs. Then, they are “lending type”. This means, we are seeking projects with IRR higher than the market rate. Since the projects are ordered in increasing costs,

A has the least initial cost. When we compare A to “Do Nothing” option, we have the increment of A having 12% IRR.

“DO NOTHING” is preferred to A, since 15% > 12%.

Next step is to compare DO NOTHING to B:

B is preferred to DO NOTHING since 18% > 15%.

Next step is to compare B and C. We cannot directly compare IRRs of B and C. We need to compare them incrementally. Since the IRR of the increment (C-B) is already given, our job is easy.

16% > 15%, meaning Project C provides everything that project B does. Not only that, the increment of C beats the market by 1%.

Final Decision: Pick C!

Numeric Problems (12 points each)

1. Twitter acquired Vine, a short-form video sharing service in 2012 for \$30 million. In 2016, Twitter would like to sell Vine. Vine is expected to make a loss of \$2 million in 2017 but start to produce a cash flow of \$4 million per year beginning in 2018. The potential buyers have an annual MARR of 10%. (hint: Vine’s cash flow is a perpetuity)

a) What is the sale price of Vine in 2016?

The value of Vine’s cash flow in 2016 = [-2M + 4M/1.1 + 4M/1.1² + 4M/1.1³ + 4M/1.1⁴ + 4M/1.1⁵.....]/1.1

= [-2M + 4M/0.1] / 1.1 = 34,545,454.5

b) Given that Twitter sells Vine at a price you found in part (a), what is the internal rate of return of Twitter’s investment on Vine?

Cash flow of Twitter’s investment on Vine:



In order to find IRR we need to equate the NPV of this cash flow to zero:

$$-30,000,000 + 34,545,454.5 / (1+r)^4 = 0$$

$$r = 3.59\%$$

2. You are given four mutually exclusive alternatives as shown in table below.

Years	A	B	C	D
0	-\$10K	-\$18K	-\$25K	-\$30K
1	\$12K	\$21K	\$29.5K	\$35K

Which alternative would you choose using IRR method?

(To get the points, student needs to look at the increments!!! Question explicitly asks for incremental Internal Rate of Return (IRR) Approach!!!)

Required: The best alternative for a MARR of 15%.

Solution:

GRADERS, NOTE THAT: We can start from any alternative and compare it against others one by one. The sequence of comparison is not important. Here, we will start from the lowest initial cost option and work our way towards the highest cost option (as suggested in one of our earlier lectures). BUT, if the student might have chosen another initial option and worked through another sequence!!!

A versus Do Nothing:

$-10 + 12/(1 + IRR_A) = 0$, $IRR_A = 20\%$, since A has "lending type cash flow", A is better than the market. Then, the defender option is A.

(B - A) Increment:

$IRR_{B-A} = (3 - 2)/(18 - 10) = 1/8 = 0.125$ or 12.5%, A is still the better option.

(C - A) Increment:

$IRR_{C-A} = (4.5 - 2)/(25 - 10) = 2.5/15 = 0.167$ or 16.7%, C is better.

(D - C) Increment:

*$IRR_{D-C} = (5 - 4.5)/(30 - 25) = 0.5/5 = 0.10$ or 10%, C is still better. **PICK C!***

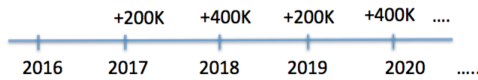
3. Atlas Inc. is funded by bonds worth of \$40 million and stocks worth of \$40 million. Bonds cost 10% and stocks cost 20% per year. Tax rate is 40%. Atlas is offered an investment opportunity today (2016) that would give \$200K in odd numbered years and \$400K in even numbered years forever. How much would Atlas pay for this investment today (2016)?

First, we need to find the MARR (the discount rate) to be applied to the cash flow of this investment opportunity:

The cost of borrowed funds: $\$40M * 10% * (1-40%) = \$2.4M \ggg 6%$

The cost of funds from stocks: $\$40M * 20% = \$8M \ggg 20%$

WACC = $6% \frac{1}{2} + 20% \frac{1}{2} = 13%$



We can think of this cash flow as a combination of two separate cash flows:

200K every year beginning 2017 + 200K every two years beginning 2018 = $200K/0.13 + 200K/0.2769 = 2,260,743.95$

where $0.2769 = (1+0.13)^2 - 1$, this is the interest rate that corresponds to two years!

(Note that cash flow can also be thought as the combination of 200K paid every 2 years and 400K paid every 2 years = $(200K/.2769)(1.13)+400K/0.2769 = 2,260,743.95$)

4. You are going to be purchasing a server for \$50,000 for your business. You have four options with the life expectancy to vary anywhere from 4 to 7 years as shown in the table below as well as the corresponding Salvage Values. The operation and maintenance cost is \$5,000 per year. The server of your choice will be replaced with the same kind of server once its life ends. If the interest rate is 10%, which server would you choose?

Life, Years	4	5	6	7
Salvage Value	\$20,000	\$20,000	\$30,000	\$30,000
Per year share of initial cost	$50,000[0.10(1.10)^4]/((1.1)^4-1) = 50,000 (A/P, 10\%, 4) = \$15,775$	$50,000[0.10(1.10)^5]/((1.1)^5-1) = 50,000 (A/P, 10\%, 5) = \$13,190$	$50,000[0.10(1.10)^6]/((1.1)^6-1) = 50,000 (A/P, 10\%, 6) = \$11,480$	$50,000[0.10(1.10)^7]/((1.1)^7-1) = 50,000 (A/P, 10\%, 7) = \$10,270$
Per year operation and maintenance costs	\$5,000	\$5,000	\$5,000	\$5,000
Per year share of Salvage Value	$20,000[0.10/((1.1)^4-1)] = 20,000 (A/F, 10\%, 4) = \$4,310$	$20,000[0.10/((1.1)^5-1)] = 20,000 (A/F, 10\%, 5) = \$3,276$	$30,000[0.10/((1.1)^6-1)] = 30,000 (A/F, 10\%, 6) = \$3,888$	$30,000[0.10/((1.1)^7-1)] = 30,000 (A/F, 10\%, 7) = \$3,162$

$EUAC_A = 15,775 + 5,000 - 4,310 = 16,465,$

$EUAC_B = 13,190 + 5,000 - 3,276 = 14,914$

$EUAC_C = 11,480 + 5,000 - 3,888 = 12,592,$

$EUAC_D = 10,270 + 5,000 - 3,162 = 12,108$

Choose D.

5. The CROC Co. is considering a new milling machine. They have narrowed the choices down to three alternatives in addition to the Null (Do nothing) alternative. The relevant data are shown in the table below.

	Economy	Regular	Deluxe
First Cost	\$75,000	\$125,000	\$220,000
Annual Benefit	\$28,000	\$43,000	\$79,000
Annual Costs	\$8,000	\$13,000	\$38,000
Salvage Value	\$3,000	\$6,900	\$16,000

All machines have a life of ten years. Using incremental rate of return analysis, which alternative should the company choose? Use a MARR of 15%.

Order the options in increasing First Costs: Null, Economy, Regular, and Deluxe.

Increment (Null up to Economy):

$$NPV = -75,000 + \{(28,000 - 8,000) (P/A, i, 10)\} + \{(3,000)(P/F, i, 10)\}.$$

$$= -75,000 + \{20,000(P/A, 15\%, 10)\} + \{3,000(P/F, 15\%, 10)\} = \$26,121.60.$$

Since, NPV is greater than \$0, incremental IRR (Internal Rate of Return) should be greater than MARR.

Economy is the defending option.

Increment (Economy up to Regular):

$$NPV = (125,000 - 75,000) + [\{(43,000 - 13,000) - (28,000 - 8,000)\} (P/A, i, 10)] + \{(6,900 - 3,000) (P/F, i, 10)\}.$$

$$= -50,000 + \{10,000 (P/A, 15\%, 10)\} + \{(3,900)(P/F, 15\%, 10)\} = \$1,154.08.$$

Since, NPV of the increment of Regular over Economy is greater than \$0 incremental IRR should be greater than MARR.

Regular is the defender.

Increment (Regular up to Deluxe):

$$NPV = - (220,000 - 125,000) + [\{(79,000 - 38,000) - (43,000 - 13,000)\} (P/A, i, 10)] + \{(16,000 - 6,900) (P/F, i, 10)\}.$$

$$= -95,000 + \{11,000(P/A, 15\%, 10)\} + \{(9,100 (P/F, 15\%, 10)\} = -\$37,541.48.$$

Since, NPV is less than \$0, incremental IRR should be less than MARR. .

Reject Deluxe.

Regular should be chosen.

6. (2 points) What is the annual effective rate of 12% APR compounded daily?

$$(1+0.12/365)^{365}-1 = 12.75\%$$

FORMULAS:

I. Cost Structure: $TC=TVC+TFC$, $ATC = TC/Q$, $MC= \Delta TC / \Delta Q$

II. Single Payment:

$$F = P(1+i)^n \quad F = P(F/P, i, n) \quad P = F(1+i)^{-n} \quad P = F(P/F, i, n)$$

III. Effective Rate:

$$\text{Effective Rate for one year: } i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

Effective Rate for more than a year (where n is the number of years):

$$i_a = \left(1 + \frac{r}{m}\right)^{m \cdot n} - 1$$

IV. Uniform Series:

$$F = A \left[\frac{(1+i)^n - 1}{i} \right] = A(F/A, i, n)$$

$$A = F \left[\frac{i}{(1+i)^n - 1} \right] = F(A/F, i, n)$$

$$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] = P(A/P, i, n)$$

$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] = A(P/A, i, n)$$

Capitalized Cost: $A = Pi$