

1. (20 points) Two companies, A and B, are operating in an oligopolistic market and are about to introduce a new product, each with their own format. Assume that each company will simultaneously decide whether to synchronize format and also whether to use freemium revenue model or direct subscription revenue model.

Freemium model requires an investment (COST) of \$10 million and subscription model requires \$5 million in case of synchronization. Otherwise costs double. Note that, for a synchronized product in the market, both companies need to choose to synchronize. The first table below summarizes the REVENUE for each company depending on their choice of revenue model in case they both choose to synchronize and the second table summarizes the similar information when at least one decides not to synchronize (all numbers are in million \$):

SYNC FORMAT:

|        |              | Firm B       |          |
|--------|--------------|--------------|----------|
|        |              | Subscription | Freemium |
| Firm A | Subscription | 68,68        | 10,70    |
|        | Freemium     | 70,10        | 35,35    |

NO SYNC FORMAT:

|        |              | Firm B       |          |
|--------|--------------|--------------|----------|
|        |              | Subscription | Freemium |
| Firm A | Subscription | 42,42        | 12,50    |
|        | Freemium     | 45,14        | 20,20    |

What is the best decision for each company in terms of synchronization and revenue model given that each wants to maximize PROFIT, neither knows exactly what the other will do but each knows the above two tables ?

*Solution:*

*Each firm has four possible actions to choose from:*

*SF: Sync and Freemium*

*SS: Sync and Subscription*

*NF: Not sync and Freemium*

*NS: Not sync and Subscription*

|        |    | Firm B |       |       |      |
|--------|----|--------|-------|-------|------|
|        |    | SS     | SF    | NS    | NF   |
| Firm A | SS | 63,63  | 5,60  | 32,32 | 2,30 |
|        | SF | 60,5   | 25,25 | 25,4  | 0,0  |
|        | NS | 32,32  | 2,30  | 32,32 | 2,30 |
|        | NF | 25,4   | 0,0   | 25,4  | 0,0  |

Note that the above table gives us the PROFIT not the revenue, therefore it incorporates the respective costs of each action. (4 points)

Assume Firm A picks SS. Then the best response from each firm consecutively would be

| <i>Firm A</i> | <i>Firm B</i> | <i>Firm A</i> | <i>Firm B</i> |
|---------------|---------------|---------------|---------------|
| SS →          | SS →          | SS →          | SS →          |

Each firm picking SS, that is "Sync and subscription" is an equilibrium.

Now assume Firm A picks SF. Checking for the best response by firm B and then by firm A...etc, gives us a second equilibrium where both firms Sync and choose Freemium.

Now assume Firm A picks NS. Checking for the best response by firm B and then by firm A...etc, gives us a third equilibrium where each firm picks Not synch and subscription.

Now assume Firm A picks NF. Checking for the best response by firm B and then by firm A...etc, lands us on one of the previously found equilibria.

Therefore, there are three equilibria. Among the three equilibria, it would be best to pick the first one (SS,SS) for each firm (10 points), yet, since the moves are simultaneous and there is no communication, this cannot be absolutely guaranteed. Then other two equilibria still survive as possible candidates. (3 points each).

Graders: If one equilibrium is correctly identified without incorporating costs, give 8 points. Full points will be obtained only when all three equilibria are mentioned and costs are incorporated.

2. (30 points) There are two companies, A and B, operating in a perfectly competitive industry. The respective cost structure for each firm is given below:

A: Total Fixed Cost = 2,900       $MC(Q) = 10Q$

B: Total Fixed Cost = 1,000       $MC(Q) = 5Q$

- a) (10 points) There are 1000 firms with the same cost structure as firm A and 500 firms with the same cost structure as B in this industry. If the market demand is given by  $P(Q) = 5,000 - 0.1Q$  and both A and B already committed to their fixed cost (they already made the purchase and payment for fixed equipment), would either produce in the short run?

*Solution* :A perfectly competitive firm produce if it can cover its variable costs in the short run. We need to know if, at the profit maximizing level of Q, either firm could cover their variable costs. Market price is not given but can be obtained by the intersection of industry demand and industry supply which is the HORIZONTAL addition of 1500 firm's individual supply curves. We need to add Quantity, not Price!!!

$$1000*(P/10)+500*(P/5) = Q_{industry}$$

Industry supply:  $P(Q) = Q/200$  (8 points)

Industry Demand:  $P(Q) = 5,000 - 0.1Q$

Market price: 238.1 (0.5 points)

Total market quantity:  $Q=47,619$  (0.5 points)

Firm A: Profit maximization requires  $238.1 = 10Q$ ,  $Q=23.81$ .  $TVC(Q) = \int 10Q = 5Q^2 + c$ . Note that the constant needs to be a part of the fixed cost therefore  $c=0$ .

Profit is  $238.1 \cdot 23.81 - 2,900 - 5 \cdot 23.81^2 = -65.42$ , this shows that even though the company is making negative profit, it can cover its variable costs. In other words, company should produce in the short run since  $238.1 \cdot 23.81 > 5 \cdot 23.81^2$ ,  $5,669.16 > 2,834.58$ .

Firm B: Profit maximization requires  $238.1 = 5Q$ ,  $Q=47.62$ .  $TVC(Q) = \int 5Q = 2.5Q^2 + c$ . Note that the constant needs to be a part of the fixed cost therefore  $c=0$ .

Profit is  $238.1 \cdot 47.62 - 1,000 - 2.5 \cdot 47.62^2 = 4,904.76$ , company should produce in the short run. (1 point)

- b) (10 points) Would either firm stay in this business in the long run given there is no new entry into the industry?

Firm A should leave the industry as it is making negative profit and firm B should stay in the long run.

- c) (10 points) Would either firm stay in this business if in the long run there is 18,000 more firms entering each having the same cost structure as firm A?

If the numbers are not calculated but just stated that higher supply will cause price to go down and make the industry less profitable for every company, then 8 points earned.

Industry supply will change  $18,000(P/10) + 200P = Q$   
 $P = (1/2,000)Q$

New equilibrium is obtained by setting the new supply equal to the original demand.

New price: 24.88

New Q: 49,751

Firm A:  $10Q=24.88$ ,  $Q=2.488$ , new profit -2,807.15

Firm B:  $5Q=24.88$ ,  $Q=4.98$ , new profit -938.09

Both firms will leave the industry in the long run.

3. (25 points) Company X is operating in a monopolistically competitive market with a total cost of  $TC(Q) = 1000 + 4Q^2$ . The demand function faced by X is given by  $P(Q) = 600 - 100Q$ .

- a) (5 points) What is the profit maximizing quantity and price for firm X?

$MC(Q) = 8Q$

$MR(Q) = 600 - 200Q$

$P=311.5$

- b) (10 points) Given that X has already produced the amount found in part a, is it possible to obtain positive profit if the company can identify 10% of its customers who have the highest willingness to pay?

At the optimum Q of 2.88 profit is -134.6. 10% of 2.88 is 0.288.

The price at this point is 571.2

Revenue from this segment is  $571.2 \cdot 0.288 = 164.51$

Revenue from the remaining segment is  $(2.88 - 0.288) \cdot 311.5 = 807.41$

Profit when price discrimination is possible:  $164.51 + 807.41 - 1,000 - 4 \cdot 2.88^2 = -61.26$ , still not profitable.

- c) (10 points) If price discrimination is not possible, at most how much would firm X for an ad campaign that would increase its demand such that the new demand line is given by  $P(Q) = 1,000 - 50Q$  ?

$MR=MC$   
 $1,000-100Q = 8Q$   
 $Q = 9.26$   
 $P = 537.04$   
 $Profit = 3,629.57$

$3,629.57+134.6 = 3,764.19$  is the maximum payment for the ad campaign.

**Each True/False question is worth 5 points. Explanation with a few sentences is necessary to get full points.**

4. TRUE/FALSE – It is not possible to price discriminate unless the firm can identify its customers' willingness to pay either individually or as a group.

*It is possible to price discriminate without identifying the willingness to pay of a certain customer. For example, by bundling two substitute commodities, A and B, offered by the same company, A and B can be priced as one item. Customer 1 may like A and 2 may like B. By purchasing the bundle, the price paid for, let's say A, is different by customers 1 and 2. When customer 1 buys the bundle, she pays most of it to A and when customer B buys the bundle, she pays most of it to B. In order to do this, company does not need to know who customer 1 or 2 is since the bundle is sold at one fixed price.*

5. TRUE/FALSE – If a firm is in the long run and decides to double its size, its total costs will less than double.

*Not necessarily. We need to know if the company is subject to economies of scale, constant returns to scale or diseconomies of scale.*

6. TRUE/FALSE – If a company has its marginal cost increasing at an increasing rate, it cannot obtain positive economic profit.

*Most marginal costs increase at an increasing rate. As long as  $TR > TC$ , company will make positive economic profit. Increasing costs do not prevent  $TR$  from being above  $TC$ , at least within a range.*

7. TRUE/FALSE – A natural monopoly is highly incentivized to use its positive economic profit for innovation.

*A natural monopoly is defined as a monopoly that enjoys the monopoly privileges due to limited demand and high initial costs. That is, there is no immediate threat to the company. No competition leads to complacency in most cases rather than innovation.*

8. TRUE/FALSE – The choice of production scale is a decision that is independent of market demand.

*Market demand is one of the crucial determinants of the scale. If demand does not exist, it does not matter if the company picked the most cost efficient scale from a production point of view.*