Midterm Examination

Your Name: *Solutions* Student Id #:

Please make sure to read the following instructions carefully and in entirety:

1. You may use your textbook plus one original handwritten cheat sheet of a two-sided page.
2. You may not use any electronic device.
3. You must show the intermediate steps in deriving your answer.
4. You must answer in the space provided in this booklet, and not use any other sheets.
4. If any sheets are missing from this exam booklet, you will get zero grade on the entire exam.

Following table to be filled by course staff only

Problem #	Maximum Score	Your Score	Comments
1	10		
2	10		
3	10		
4	15		
5	15		
TOTAL	60		

IMPORTANT

Please do not tear off or remove any pages from this exam booklet. You must return all the pages of this booklet, and failure to do so will void your exam.

This solution file available at http://goo.gl/U9Bzdo

Problem #1 [5 * 2 = 10 points]

Place an X to the left of the correct choice in the following multiple choice questions. There is space on the next page for you to work out the answers in case you need it.

a) Simplify the Boolean expression $((A \lor B \lor C) \land \neg(D \lor E)) \lor ((A \lor B \lor C) \land (D \lor E))$ and choose the best answer.

1. $A \lor B \lor C$ **CORRECT**
2. $D \lor E$
3. $\neg A \land \neg B \land \neg C$
4. $\neg D \land \neg E$
5. None of the above

b) Which of the following relationships represents the dual of the Boolean property $x \lor (\neg x \land y) = x \lor y$?

1. $\neg x \land (x \lor \neg y) = \neg x \land \neg y$
2. $x \land (\neg x \land y) = x \land y$
3. $x \land \neg x \lor y$
4. $\neg x \land (x \land \neg y) = \neg x \land \neg y$
5. $x \land (\neg x \lor y) = x \land y$ **CORRECT**

c) Given the function $f(X,Y,Z) = (X \land Z) \lor (Z \land (\neg X \lor (X \land Y)))$, the equivalent most simplified Boolean representation for $f(X,Y,Z)$ is:

1. $Z \lor (Y \land Z)$
2. $Z \lor (X \land Y \land Z)$
3. $X \land Z$
4. $X \lor (Y \land Z)$
5. None of the above **CORRECT:** $f(X,Y,Z) = Z$

d) Simplification of the Boolean expression $(\neg(A \lor B) \land \neg(C \lor D \lor E)) \lor \neg(A \lor B)$ yields which of the following results?

1. $A \lor B$
2. $\neg A \land \neg B$ **CORRECT**
3. $C \lor D \lor E$
4. $\neg C \land \neg D \land \neg E$
5. $\neg A \land \neg B \land \neg C \land \neg D \land \neg E$

e) Given that $F = (\neg A \wedge \neg B) \vee \neg C \vee \neg D \vee \neg E$, which one of the following represents the correct expression for $\neg F$?

1. $\neg F = A \vee B \vee C \vee D \vee E$
2. $\neg F = A \wedge B \wedge C \wedge D \wedge E$
3. $\neg F = A \wedge B \wedge (C \vee D \vee E)$
4. $\neg F = (A \wedge B) \vee \neg C \vee \neg D \vee \neg E$
5. $\neg F = (A \vee B) \wedge C \wedge D \wedge E$ **CORRECT**

Problem #2 [10 points]

Determine the minimum-cost PoS expression and correspond circuit using NOR gates for the function $\sum m(4,6,8,10,11,12,15)+D(3,5,7,9)$. Assume the input variables are labeled x3, x2, x1, and x0, and the minterms are numbered with x3 in the most significant position and x0 in the least significant position (e.g. minterm 1 corresponds to the product term $\neg x3 \wedge \neg x2 \wedge \neg x1 \wedge x0$). Show all the necessary intermediate steps. Note that $D(\,)$ represents the don't care set.

Answer #2:

The problem requires NOR gates, which indicates that we seek a PoS form.

APPROACH 1: We will develop a SoP for the complement of the function and then use De Morgan's theorem to get the PoS for the original function.

The complement of the function is $\sum m(0,1,2,13,14)+D(3,5,7,9)$.

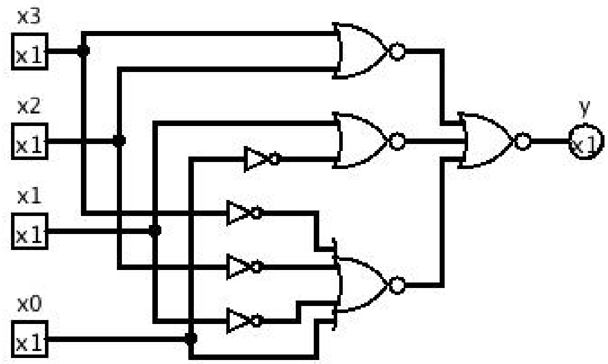
This gives the following K-Map, with the minimal cover shown:

x3, x2 \ x1, x0	00	01	11	10
00	1	1	x	1
01	0	x	x	0
11	0	1	0	1
10	0	x	0	0

The K-Map cover yields the following expression for the complement of the function:
$(\neg x3 \wedge \neg x2) \vee (\neg x1 \wedge x0) \vee (x3 \wedge x2 \wedge x1 \wedge \neg x0)$

Using duality theorem which states that the complement of the dual of a function evaluated over the complements of the inputs is the same as the original function over the original inputs, we get the following expression for the original function
$(x3 \vee x2) \wedge (x1 \vee \neg x0) \wedge (\neg x3 \vee \neg x2 \vee \neg x1 \vee x0)$ which using De Morgan's theorem can be rewritten as $\neg(\neg(x3 \vee x2) \vee \neg(x1 \vee \neg x0) \vee \neg(\neg x3 \vee \neg x2 \vee \neg x1 \vee x0))$

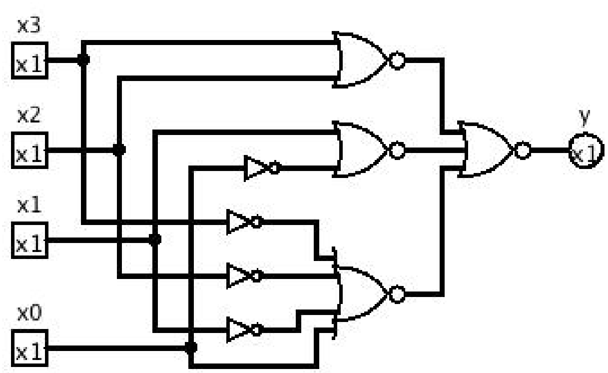
Answer #2 (contd.):



APPROACH 2: We can directly work with the K-Map of the original function but focus on entries with 0 and directly go to product of sum form.

x3, x2 \ x1, x0	00	01	11	10
00	0	0	x	0
01	1	x	x	1
11	1	0	1	0
10	1	x	1	1

$(x3 \vee x2) \wedge (x1 \vee \neg x0) \wedge (\neg x3 \vee \neg x2 \vee \neg x1 \vee x0)$ which using De Morgan's theorem can be rewritten as $\neg(\neg(x3 \vee x2) \vee \neg(x1 \vee \neg x0) \vee \neg(\neg x3 \vee \neg x2 \vee \neg x1 \vee x0))$ and yields the circuit



Problem #3 [10 points]

Implement the following function with inputs $w1$, $w2$, $w3$, and $w4$:

$$f(w1, w2, w3, w4) = (w1 \wedge w2 \wedge w4) \vee (w1 \wedge w2) \vee (w1 \wedge w3) \vee (w1 \wedge w4) \vee (w3 \wedge w4)$$

You have two options for this question.

Easy half credit option (i.e. maximum score you'd receive will be 5 out of 10): implement using an 8-to-1 multiplexer and as few other gates as possible from the following types of gates: inverter, 2-input NAND, and 2-input NOR.

Harder full credit option (i.e. maximum score you'd receive will be 10): implement using a 4-to-1 multiplexer and as few other gates as possible from the following types of gates: inverter, 2-input NAND, and 2-input NOR.

Answer #3:

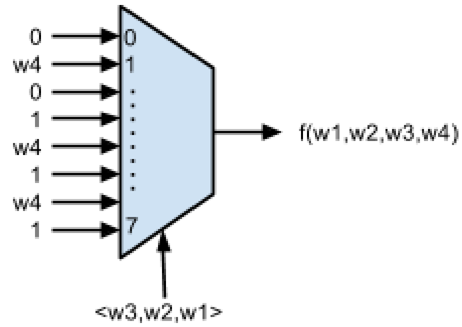
Easy half credit option:

Our strategy is to use 3 of the 4 inputs to control the mux, and use the remaining one to influence the data inputs to the mux. Several different solutions are possible depending on which inputs you select. Let us say we use $< w3, w2, w1 >$ as the control input $sb[2:0]$ of the 8-to-1 mux. Then the following table helps us derive how the data input lines of the mux should be connected.

$w3$	$w2$	$w1$	$f(w1, w2, w3, 0)$	$f(w1, w2, w3, 1)$	$f(w1, w2, w3, w4)$
0	0	0	0	0	0
0	0	1	0	1	$w4$
0	1	0	0	0	0
0	1	1	1	1	1
1	0	0	0	1	$w4$
1	0	1	1	1	1
1	1	0	0	1	$w4$
1	1	1	1	1	1

Answer #3 (contd.):

This yields the following circuit which only needs the 8-to-1 mux:



Harder full credit option

Our strategy is to use 2 of the 4 inputs to control the mux, and use the remaining one to influence the data inputs to the mux. Several different solutions are possible depending on which inputs you select. Let us say we use $< w2, w1 >$ as the control input $sb[1:0]$ of the 4-to-1 mux. Then the following table helps us derive how the data input lines of the mux should be connected.

$f(w1,w2,w3,w4) = (w1 \wedge w2 \wedge w4) \vee (w1 \wedge w2) \vee (w1 \wedge w3) \vee (w1 \wedge w4) \vee (w3 \wedge w4)$

$w2$	$w1$	$f(w1,w2,0,0)$	$f(w1,w2,1,0)$	$f(w1,w2,0,1$	$f(w1,w2,1,1)$	$f(w1,w2,w3,w4)$
0	0	0	0	0	1	$w3 \wedge w4$
0	1	0	1	1	1	$w3 \vee w4$
1	0	0	0	0	1	$w3 \wedge w4$
1	1	1	1	1	1	1

This yields the following circuit which needs the 4-to-1 mux, one 2-input NAND, one 2-input NOR, and one inverters. Note that your circuit may look different if you selected different signals as mux control inputs.